NUMERICAL METHODS FOR NONLINEARLY DAMPED OSCILLATORS

By

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IN THE LOVING MEMORY

OF MY GRANDPARENTS.......

CERTIFICATE

This is to certify that the thesis entitled "NUMERICAL METHODS FOR NONLINEARLY DAMPED OSCILLATORS" which is being submitted by Mr. Rajive Kumar for the award of the degree, DOCTOR OF PHILOSOPHY in MATHEMATICS, to the Indian Institute of Technology, Delhi, is a bonafide record of research work done under our guidance and supervision.

The thesis has reached the standard fulfilling the requirements of the regulations relating to the degree. The results obtained in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

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SYNOPSIS

The general second order initial value problems

$$\begin{aligned} & y'' = f(t,y,y'), \quad t > t_o \\ & y(t_o) = y_o, \quad y'(t_o) = y'_o \end{aligned} \tag{1}$$

having oscillating solutions arise in a variety of physical problems. The use of classical numerical methods for solving (1) restricts the choice of the step size to small value because of stability constraints.

The modification of classical numerical methods to simulate the oscillating phenomena has been discussed by several researchers. These modifications are based on the philosophy of forcing the methods to have trigonometric order along with polynomial order. Such methods depend on ω, the frequency associated with the given initial value problem. However, these modifications are restricted to the initial value problems (1) which are independent of y'.

In this thesis, we have developed some efficient single-step and two-step numerical methods having trigonometric and polynomial orders for the direct integration of the general second order initial value problem (1). The thesis consists of six chapters. A brief description of the contents of each chapter is as follows:

CHAPTER 1: SURVEY OF NUMERICAL METHODS FOR GENERAL SECOND ORDER DIFFERENTIAL EQUATIONS

In this chapter a general discussion of numerical methods for solving the general second order initial value problem (1) is given. Some advantages and disadvantages of these methods are pointed out.

CHAPTER 2 : A SIXTH ORDER MODIFICATION OF THE STIEFEL-BETTIS METHOD FOR NONLINEARLY DAMPED OSCILLATORS*

A two-step implicit method of trigonometric order one and polynomial order four has been obtained for the general second order initial value problem (1). The method has the local truncation error of $O(h^8)$. The method depends on ω, the frequency associated with the given initial value problem. The numerical results, of three problems are listed to illustrate the method. The numerical results demonstrate that for problems, with large frequency, the methods having trigonometric order produce accurate results. This method is a modification of the Stiefel-Bettis method obtained for the second order initial value problem $y'' = f(t,y)$.

*The results contained in this chapter have been published in "Computer Methods in Applied Mechanics and Engineering", 72(1989), pp. 187-193.

CHAPTER 3: EXPLICIT METHODS FOR NONLINEARLY DAMPED OSCILLATORS*

Two-step explicit methods of trigonometric order one and polynomial orders two and four have been obtained for solving (1). The methods depend on ω, the frequency associated with the given initial value problem. The numerical results of three problems are given to illustrate the method. Numerical results demonstrate that explicit methods are both economical and accurat

CHAPTER 4: BLOCK EXPLICIT METHODS FOR NONLINEARLY DAMPED OSCILLATORS**

n-point block explicit methods for the direct integration of (1) have been obtained for n = 3 to 6. The methods have trigonometric order one and polynomial order n-2. These methods provide solutions in blocks of n points rather than one point at a time. The values of only y_0 and y_0' are needed to get the block started. These methods are computationally efficient in the sense that the number of function evaluations is considerably reduced by increasing the number of points in the block. The methods depend on ω, the frequency associated with the given initial value problem. The numerical results of five problems are given to illustrate the method.

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* A part of the results contained in this chapter have been accepted for publication in JMPS.

**A part of the results contained in this chapter have been accepted for publication in the Indian Journal of Pure and Applied Mathematics.

CHAPTER 5: SUPERSTABLE IMPLICIT RUNGE-KUTTA METHODS FOR SECOND ORDER INITIAL VALUE PROBLEMS*

Using the well known properties of the s-stage implicit Runge-Kutta methods for the first order differential equations, singlestep methods of arbitrary order have been obtained for the direct integration of the general second order initial value problem (1). The methods, of second, fourth and sixth orders, based on Gauss-Legendre, Lobatto IIIA and Lobatto IIIB nodes are listed. These methods when applied to the test equation $y'' + 2\alpha y' + \beta^2 y = 0$, $\alpha, \beta \geq 0$, $\alpha+\beta > 0$ are shown to be superstable with the exception of a finite number of isolated values of βh. We have also obtained almost superstable Obrechkoff type methods of order 2s involving (s-1)th order derivative of f for the direct integration of (1). It has been established that any numerical method, for a first order differential equation, whose stability function is a (s,s) Pade' approximation, when applied to the general second order initial value problem (1) provides an almost superstable method of order 2s. The numerical results of three problems are given to illustrate the method.

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*The results contained in this chapter have been published in BIT, 29(1989), p. 518-526.

CHAPTER 6: COMPARATIVE STUDY OF THE METHODS DEVELOPED IN THE THESIS

In this chapter we have compared the numerical results of three problems obtained by using implicit and explicit methods of trigonometric order one and polynomial order four with those obtained by using the methods having polynomial order six. Based on these numerical results we conclude that for problems with small frequency, the superstable method produces most accurate results whereas for problems having large frequency methods having trigonometric order produce more accurate results.

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