FINITE ELEMENT ANALYSIS OF FLUID FLOWS AT LOW AND MODERATE REYNOLDS NUMBERS : APPLICATIONS TO BLOOD FLOW

By

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CERTIFICATE

This is to certify that the thesis entitled "Finite Element Analysis of Fluid Flows at Low and Moderate Reynolds Numbers: Applications to Blood Flow," being submitted by Qamare Hasan to the Indian Institute of Technology, Delhi (India) for the award of the degree of Doctor of Philosophy in Applied Mechanics Department is a record of bonafide research work carried out by him under our supervision and guidance. The thesis work, in our opinion, has reached the standard fulfilling the requirements for the Doctor of Philosophy Degree. The research report and the results presented in this thesis have not been submitted in part or in full to any other University or Institute for the award of any degree or diploma.

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ABSTRACT

The fluid flow at low and moderate Reynolds numbers is encountered in many physiological situations involving blood flow in different systems. In physiological applications where complicated boundary conditions and irregular flow geometry are to be incorporated numerical techniques based on finite difference methodology become complex and expensive. In this thesis finite element formulations are presented for analysing the fluid flows in such situations. Isoparametric ring elements have been used with velocity and pressure as nodal variables. For analysing creeping flows two formulations, namely, Galerkin weighted residual and variational methods have been used. Separate formulation for the solution of pressure field based on finite element method has also been presented and tested. These formulations have been compared with respect to the accuracy of the results computed, by applying them to analyse the flow field in the entry region of a circular tube.

The blood flow through capillaries incorporating the actual deformed shapes of the erythrocytes (RBC) has been analysed and the effect of RBC interaction has been studied by using the creeping flow analysis program based on Galerkin weighted residual method. The computed results for velocity field, additional pressure drop, shear stresses on the cell surfaces as well as on capillary wall and the tension in the cell membrane are compared with the available experimental data.

Finite element formulations for solving the full Navier-Stokes equations using two different iterative methods, namely, the Crank Nichlson Scheme with variable coefficient (Relaxation Method) and perturbation method have been developed. The two methods have been compared with respect to (i) accuracy of the results obtained (ii) computer time taken, by applying them to analyse the flow field in the entry region of a circular tube at moderate Reynolds numbers. The methodology using perturbation iterative method has further been applied to analyse the developing flow in the entry region of a tube with different inlet velocity profiles. The method has also been applied to analyse the velocity and pressure fields in tubes with constrictions. The flow field in the region of a stenosis in a tube with varying degrees of blockage has been computed at various Reynolds numbers. The flow through a venous valve has also been analysed at various Reynolds numbers in the range of 0-40. The computed results have been compared with the existing data in the literature.

It has been observed through the various results computed in the present work, that finite element method is helpful in analysing the flow where complicated boundary conditions and irregular flow geometries are involved.

CONTENTS

NOMENCLATURE

r	Radial Co-ordinate
z	Axial Co-ordinate
u, w	Radial and Axial Components of Velocity
U	Dimensionless radial velocity
W	Dimensionless axial Velocity
W_{CL}	Non-dimensional Centreline Velocity
W_c	Relative Cell Velocity
$\overline{W}$	Average Axial Velocity
R, Z	Dimensionless Radial and Axial Co-ordinates
D	Diameter of the Tube
L	Length of the Tube.
L_e	Entry Length
℄.	Centre Line.
p	Pressure
$\langle p \rangle$	Mean Pressure
p_i, p_o	Inside and Outside Pressures
R_o	Radius of the Tube
Re	Reynolds Number
Δp	Pressure Drop in the Tube
e	Strain Rate
Δp^*	Additional Pressure Drop
$\Delta p(Pois)$	Pressure Drop due to Poiseuille Flow
$\hat{n}$	Unit normal Vector
ξ, η	Local Co-ordinates

$[K]$	Element Characteristic Matrix
$\{\phi\}$	Matrix for Nodal Variables
$\{F\}$	Matrix of Forcing Functions
$[J]$	Jacobian Matrix
$[\cdot]^{-1}$	Inverse of Matrix
$[\ ]^{T}$	Transpose of Matrix
det $\|J\|$	Determinant of Matrix J
N, N^{p}	Shape Functions for Velocity and Pressure
μ	Dynamic Viscosity
ρ	Density of the Fluid
μ_r	Apparent Relative Viscosity
τ	Shear Stress
τ_w	Wall shear stress
τ_{cs}	Shear Stress at the Cell Surface
τ_w(Pois)	Wall Shear Stress due to Poiseuille Flow
D_c	Diameter of the Erythrocyte
$\hat{i}$, $\hat{j}$, $\hat{k}$	Unit Vectors along Global Co-ordinates
c.p.	Centi Poise
Vc	Volume of the Cell _(μm^3)
ψ	Stream Function
$\bar{H}$	Average Tube Hematocrit
Ho	Cupmixing Hematocrit
$\dot{Q}$	Volume Rate of Flow
d	Diameter at the narrowest Section of Stenotic and Valved Vessels

d/D	Diameter Ratio
T	Tension in the Membrane
θ	Angular Position
R_f	Relaxation Factor
ϵ	Perturbation Coefficient
μm	Dimension in micron (1 micron = 10^{-6}m)