RECOVERY OF STRESS GRADIENTS AND THEIR USE FOR OPTIMUM MESH DESIGN

By

VIJAY KUMAR CHOPRA

THESIS SUBMITTED TO THE
INDIAN INSTITUTE OF TECHNOLOGY, DELHI
FOR THE AWARD OF DEGREE

DOCTOR OF PHILOSOPHY

Department of Applied Mechanics
INDIAN INSTITUTE OF TECHNOLOGY, DELHI

JULY, 1995

CERTIFICATE

This is to certify that the thesis entitled " Recovery of Stress Gradients and their use for Optimum Mesh Design " being submitted by Mr. Vijay Kumar Chopra to the Indian Institute of Technology Delhi for the award of the Degree of Doctor of Philosophy, is a record of bonafide research work carried by him under my supervision and guidance.

The results contained in this thesis have not been submitted in part or in full to any other University or Institute for the award of any degree or diploma.

D. K. Sehgal

(Dr. D. K. Sehgal)
Assistant Professor,
Department of Applied Mechanics,
Indian Institute of Technology, Delhi,
New Delhi - 110 016.

ACKNOWLEDGEMENTS

The author wishes to express his most sincere thanks and his deep sense of gratitude to Dr. D. K. Sehgal for his invaluable guidance throughout the course of the study. His constant attention and encouragement at every step enabled the author to give the dissertation its present form.

The author is thankful to the Faculty of Engineering and Technology, Jamia Millia Islamia, New Delhi, for sponsoring him for higher studies under the Quality Improvement Programme of the Government of India.

The author also wishes to thank all the faculty members of the Applied Mechanics Department for their worthy suggestions.

The author would like to express his sincere thanks to Mr. Sriram Hegde, Mr. G.V.V. Ravikumar, Mr.R.Marimuthu, Mr. Velmurgan, Mr. A. Dutta and Mr. K. K. Pathak for their support.

The author also acknowledges the help and assistance given to him by all other persons whom he is unable to mention here individually .

Finally, the author expresses his deep appreciation for the understanding and spontaneous co-operation extended by his wife Kusum and sons Vikram and Karan which made it possible for him to complete his research work.

ABSTRACT

The objectives of the present study were

a) to develop a generalised procedure for the determination of higher order gradients

b) to determine the optimum locations for the calculation of stress gradients

c) to use the stress gradients for the optimum design of FEM meshes and their adaptive refinement

For obtaining the stress gradients, we can differentiate the stress components as obtained from routine FEM analysis. This method, which has been designated as the Direct Method, cannot be used in the case of linear elements as the second derivatives become zero. It is also quite cumbersome to build up higher derivatives by this method. In order to overcome these difficulties an alternate technique is proposed which is based on average nodal stresses and the iso-parametric concept. Using this technique we can build up the derivatives to any desired order. Even linear elements do not pose any difficulty.

In order to identify the best possible locations where the gradients could be obtained with a greater degree of confidence, a generalised procedure is used to obtain Optimal locations for stress gradients in the case of the linear and quadratic quadrilateral elements. It is observed that the origin is the best location within the element for the determination of stress gradients. If these are required to be calculated at the boundary of the element then the mid-points of the sides are the best locations. These locations are verified by performing numerical experimentation on five structural engineering problems as given below :

(i) Deep beam (ii) Plate with a hole

(iii) Circular plate with a hole (iv) Wall bracket

(v) Gravity dam

Only medium refined meshes of either linear or quadratic elements are used. These results are compared against those obtained by using the same number of 25-noded quartic order Lagragian quadrilateral elements, the latter results being treated as standard. Then the root mean square errors in gradients are calculated. These are plotted with respect to their corresponding locations within the element in order to obtain points where errors are less. There is no appreciable difference in the behaviour of serendipity and Lagrangian elements as far as the optimal locations for stress gradients are concerned. The above results are also confirmed by the proposed alternate method for the determination of stress gradients.

At the moment there is no appropriate procedure available for FEM discretisation. Meshes, as they are drawn, depend upon the user's skill. This area is considered as the Finite Element Art. However the FEM mesh plays a crucial role for the accuracy of the final solution. Keeping this in mind a study is conducted to understand the behaviour of the parameters which are important w.r.t. the criteria for failure and which vary smoothly within the domain. The parameters considered are distortion energy, strain energy, max. shear stress, octahedral shear stress, R.M.S. value of stress components and their gradients. Contours showing their variation are plotted for the deep beam problem. The contours based on the R.M.S. value of stress gradients compared well with the optimum mesh obtained as a result of numerical experimentation. Hence this parameter is selected for further investigation.

The optimal meshes based on the stress gradient contours of linear/quadratic elements are obtained and analysed for the following cases :

i) Plate with a hole ii) Wall bracket

For the purpose of ascertaining the performance of these proposed meshes the results obtained from the analysis are compared with the results as given by standard meshes. It has been found that the results improve substantially if the meshes are based on stress gradient contours. The results of these meshes are used to replot contours. The meshes based on the replotted contours showed further improvement in results.

The optimal mesh design is also attempted by using contours of stress gradients based on higher order 25-noded element. Meshes are prepared for all the five problems as mentioned before. These meshes are analysed using either linear or quadratic elements and the errors computed . It is observed that as the contour interval (percentage gradient drop) decreases, the results improve. Also we are able to obtain very good results even with a substantial decrease in the number of elements.

CONTENTS

Description	Page

CERTIFICATE		i
ACKNOWLEDGEMENTS		ii
ABSTRACT		iii
LIST OF FIGURES		vi
LIST OF TABLES		x
NOMENCLATURE		xi
CHAPTER-1	RECOVERY OF STRESS GRADIENTS	1
1.1	Introduction	1
1.2	Procedure For Calculation of Stresses in FEM	1
1.3	Optimal Stress Locations In Finite Element Models	3
1.4	Barlow's Procedure	4
1.4.1	Optimal Stress Locations For Quadratic Bar Element	4
1.5	Generalised Procedure For Obtaining Optimal Stress Locations	6
1.6	Stress Gradients	6
1.7	The Present Work	7
1.7.1	Procedure For Obtaining Stress Derivatives	7
1.7.2	Determination of Optimal Locations for Stress Gradients	8
CHAPTER-2	DETERMINATION OF STRESS GRADIENTS	9
2.1	Introduction	9
2.2	Direct Method For The Calculation Of Stress Gradients	9
2.3	Modifications In The FEM Package For The Direct Determination Of Stress Derivatives	18
2.4	Proposed Alternate Method For The Calculation Of Stress Gradients	21
2.5	Modifications In The FEM Package For The Alternate Method	23
2.6	Conclusions	24

Description	Page

CHAPTER-3 25

THEORETICAL DETERMINATION OF OPTIMAL LOCATIONS FOR STRESS GRADIENTS FOR LINEAR QUADRILATERAL ELEMENTS

3.1 Introduction 25
3.2 Determination Of Optimal Locations For Stress Gradients 25
3.3 Discussion Of The Results 30
3.4 Conclusions 31

CHAPTER-4 34

NUMERICAL DETERMINATION OF OPTIMAL LOCATIONS FOR STRESS GRADIENTS FOR LINEAR QUADRILATERAL ELEMENTS

4.1 Introduction 34
4.2 Locations For The Computation Of Stress Gradients 34
4.2.1 Set 1 34
4.2.2 Set 2 35
4.2.3 Set 3 35
4.3 Lagrangian Quartic 25-Noded Quadrilateral Element 35
4.3.1 Shape Functions For Quartic 25-Noded Quadrilateral Element 35
4.4 Error Estimation 37
4.5 Problems Selected For Numerical Experimentation 38
4.5.1 Deep Beam 38
4.5.2 Plate With a Hole 39
4.5.3 Circular Plate With a Hole 39
4.5.4 Wall Bracket 39
4.5.5 Gravity Dam 39
4.6 Results 40
4.6.1 Result Of Deep Beam 40
4.6.2 Result Of Plate With a Hole 40
4.6.3 Result Of Circular Plate With a Hole 41
4.6.4 Result Of Wall Bracket 41
4.6.5 Result Of Gravity Dam 42
4.7 Discussion Of The Results 42
4.8 Conclusions 43

Description **Page**

CHAPTER-5 **62**

THEORETICAL DETERMINATION OF OPTIMAL LOCATIONS FOR STRESS GRADIENTS FOR QUADRATIC QUADRILATERAL ELEMENTS

5.1 Introduction 62
5.2 Determination Of Optimal Locations For Stress Gradients For The 9-Noded Lagrangian Quadrilateral Element 62
5.3 Determination Of Optimal Locations For Stress Gradients For The 8-Noded Serendipity Quadrilateral Element 69
5.4 Discussion Of The Results 76
5.4.1 Discussion For 9-Noded Lagrangian Element 76
5.4.2 Discussion For 8-Noded Serendipity Element 77
5.5 Conclusions 77

CHAPTER-6 **81**

NUMERICAL DETERMINATION OF OPTIMAL LOCATIONS FOR STRESS GRADIENTS FOR QUADRATIC QUADRILATERAL ELEMENTS

6.1 Introduction 81
6.2 Estimation Of Errors 81
6.3 Numerical Experimentation Results 81
6.3.1 Results Of Deep Beam 81
6.3.2 Results Of Plate With a Hole 82
6.3.3 Results Of Circular Plate a Hole 83
6.3.4 Results Of Wall Bracket 84
6.3.5 Results Of Gravity Dam 85
6.4 Discussion 85
6.5 Conclusions 86

CHAPTER-7 OPTIMAL MESH DESIGN - A REVIEW **107**

7.1 Introduction 107
7.2 Adaptive Mesh Design - A Brief Survey 107
7.3 Adaptive Refinement Methodologies 109
7.4 The Z-Z Procedure For Adaptive Mesh Refinement 109
7.5 The Drawbacks Of The Adaptive Refinement Procedures 111
7.6 The Present Work 112

Description		Page
CHAPTER-8	**OPTIMAL MESH DESIGN**	**113**
8.1	Introduction	113
8.2	Determination Of Optimum Mesh By Numerical Experimentation	113
8.2.1	Behaviour Of Horizontal Divisions	113
8.2.2	Behaviour Of Vertical Divisions	115
8.3	Theoretical Determination Of Optimum Mesh	116
8.3.1	Distortion Energy	117
8.3.2	R.M.S. (Root Mean Square) Value Of The Derivatives Of The Distortion Energy	118
8.3.3	Strain Energy	119
8.3.4	R.M.S. Value Of The Derivatives Of The Strain Energy	120
8.3.5	Max. Shear Stress	120
8.3.6	R.M.S. Value Of The Derivatives Of The Max. Shear Stress	120
8.3.7	Octahedral Shear Stress	121
8.3.8	R.M.S. Value Of The Derivatives Of The Octahedral Shear Stress	122
8.3.9	R.M.S. Value Of The Stress Components	123
8.3.10	R.M.S. Value Of The Stress Gradients	123
8.4	Contour Plots Showing Variation Of Parameters	124
8.4.1	Discussion Of Results	124
8.5	Conclusions	125
CHAPTER-9	**OPTIMAL MESH DESIGN USING LINEAR AND QUADRILATERAL ELEMENTS**	**140**
9.1	Introduction	140
9.2	Stress Gradient Contours	140
9.3	Designing The Optimal Mesh	140
9.4	Optimal Mesh Design Using Linear Elements	141
9.4.1	Plate With a Hole	141
9.4.2	Wall Bracket	141
9.5	Optimal Mesh Design Using Quadratic Elements	142
9.5.1	Plate With a Hole	142
9.5.2	Wall Bracket	142
9.6	Error Estimation	142
9.7	Discussion Of Results	144
9.7.1	Results For Linear Elements	144
9.7.2	Results For Quadratic Elements	144
9.8	Adaptive Improvement Of The Results	145
9.9	Conclusions	146

	Description	Page
CHAPTER-10	OPTIMAL MESH DESIGN USING HIGHER ORDER ELEMENT	161
10.1	Introduction	161
10.2	Optimal Mesh Design Using Higher Order Element	161
10.2.1	Results of Deep Beam	162
10.2.2	Results of Plate With a Hole	163
10.2.3	Results of Circular Plate With a Hole	163
10.2.4	Results of Wall Bracket	164
10.2.5	Results of Gravity Dam	165
10.3	Mathematical Interpretation Of The Technique	166
10.4	Conclusions	167
CHAPTER-11	FINAL CONCLUSIONS	200
11.1	Determination Of Stress Gradients	200
11.2	Theoretical Determination Of Optimal Locations For Stress Gradients	200
11.3	Numerical Determination Of Optimal Locations For Stress Gradients	201
11.4	Optimal Mesh Design Using Stress Gradients	201
11.5	Significant New Contributions	202
11.6	Future Scope	203
REFERENCES		204
BRIEF BIO-DATA OF THE AUTHOR		208