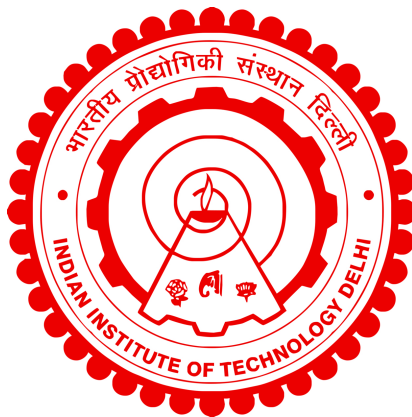


NONLINEAR HIERARCHICAL CONTROL OF QUADROTOR AND QUADROTOR MANIPULATION SYSTEM

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DEPARTMENT OF ELECTRICAL ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY DELHI

JANUARY 2024

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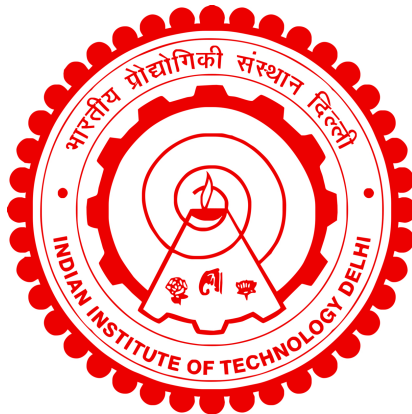
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Submitted

in partial fulfillment of the requirements of the degree of Doctor of Philosophy

to the



**INDIAN INSTITUTE OF TECHNOLOGY
DELHI**

JANUARY 2024

Dedicated to friends and family...

Certificate

This is to certify that the thesis entitled **Nonlinear Hierarchical Control of Quadrotor and Quadrotor Manipulation System** submitted by **Ranjan Dasgupta** to the Indian Institute of Technology Delhi, for the award of the Degree of **Doctor of Philosophy**, is a record of the bona fide research work carried out by him under my supervision and guidance. The thesis has reached the standards fulfilling the requirements of the regulations relating to the degree. The results contained in this thesis have not been submitted either in part or in full to any other University or Institute for the award of any degree or diploma to the best of my knowledge.

Prof. Shubhendu Bhasin
Department of Electrical Engineering,
Indian Institute of Technology Delhi.
(Supervisor)

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Ranjan Dasgupta

Abstract

This thesis deals with the design and analysis of stabilizing singularity free hierarchical nonlinear control for a quadrotor system and a quadrotor equipped with a robotic arm - quadrotor manipulation system (QMS). A two-fold contribution is made to nonlinear analysis and synthesis of control hierarchy applied to a quadrotor maneuver and subsequently to a QMS under free-light conditions or a QMS undergoing physical interactions with the environment.

It is well understood that the under-actuated quadrotor system involves position and attitude dynamics in cascade. The initial works study the hierarchical nonlinear control strategy of a quadrotor which requires reference attitude extraction from the designed position control input to derive the attitude control law. However, they typically do not provide the singularity-free desired attitude calculation unless the position control input is bounded. It is found that the widely used saturated position control designs are not smooth enough for the subsequent attitude control development based on the hierarchical procedure. Moreover, attitude singularity often occurs during attitude tracking if the pitch constrained attitude control design is not enforced. In this case, the computed control input may be unbounded. Since the attitude controller doesn't have an infinite bandwidth, the attitude tracking error introduces a strong nonlinear coupling between the position and attitude dynamics. Its effect on stability of the overall system is typically not analyzed in the current literature. In this thesis, a saturation control scheme using hyperbolic tangent function is designed for the position control in order to ensure nonsingular command attitude extraction. A pitch constrained attitude control using barrier Lyapunov function (BLF) with an initial condition constraint is proposed to obviate the problem of attitude singularity during attitude tracking.

Another concern is that tracking control of a quadrotor is degraded on interaction with the physical environment using an attached robotic arm / manipulator. This happens due to changes in the inertia parameters. This work introduces adaptive hierarchical attitude control in order to establish robust tracking performance in presence of parameter uncertainties. A multistage constructive procedure with rigorous stability analysis, considering the effect of nonlinear coupling between the two

subsystems, has been synthesized at every stage of nonlinear hierarchical control design of a quadrotor system.

Furthermore, the main effects that appear and make the dynamic behavior of the QMS different from the standard quadrotor, is the dynamic reaction forces and torques generated by the movement of the arm as well as impact forces while making contact with the physical object. Since QMS is a complex multibody system, this part of the work extends the previously designed nonlinear hierarchical control and applied on position, attitude and manipulation subsystems together considering its coupled dynamic behaviour and environment interaction aspects. The pitch constraint attitude control design is adopted to address the singularity problem which may otherwise severely affect the stability of the QMS while the manipulator end effector trying to make contact with the stationary or dynamic object via impact collision.

सार

यह थीसिस एक क्वाड्रोटर सिस्टम और एक रोबोटिक आर्म - क्वाड्रोटर हेरफेर सिस्टम (क्यूएमएस) से सुसज्जित क्वाड्रोटर के लिए विलक्षणता मुक्त पदानुक्रमित नॉनलाइनियर नियंत्रण को स्थिर करने के डिजाइन और विश्लेषण से संबंधित है। क्वाड्रोटर पैतरेबाज़ी पर लागू नियंत्रण पदानुक्रम के गैर-रेखीय विश्लेषण और संश्लेषण में दो गुना योगदान दिया जाता है और बाद में फ्री-लाइट स्थितियों के तहत क्यूएमएस या पर्यावरण के साथ भौतिक बातचीत से गुजरने वाले क्यूएमएस पर लागू किया जाता है।

यह अच्छी तरह से समझा जाता है कि अंडर-एक्टिवेटेड क्वाड्रोटर प्रणाली में कैस्केड में स्थिति और दृष्टिकोण की गतिशीलता शामिल है। प्रारंभिक कार्य एक क्वाड्रोटर की पदानुक्रमित गैर-रेखीय नियंत्रण रणनीति का अध्ययन करते हैं जिसके लिए रवैया नियंत्रण कानून प्राप्त करने के लिए डिज़ाइन किए गए स्थिति नियंत्रण इनपुट से संदर्भ रवैया निष्कर्षण की आवश्यकता होती है। हालाँकि, वे आम तौर पर विलक्षणता-मुक्त वांछित रवैया गणना प्रदान नहीं करते हैं जब तक कि स्थिति नियंत्रण इनपुट सीमित न हो। यह पाया गया है कि व्यापक रूप से उपयोग किए जाने वाले संतृप्त स्थिति नियंत्रण डिज़ाइन पदानुक्रमित प्रक्रिया के आधार पर बाद के रवैया नियंत्रण विकास के लिए पर्याप्त सुचारू नहीं हैं। इसके अलावा, यदि पिच नियंत्रित रवैया नियंत्रण डिजाइन लागू नहीं किया जाता है, तो रवैया ट्रेकिंग के दौरान रवैया विलक्षणता अक्सर होती है। इस मामले में, परिकल्पित नियंत्रण इनपुट असीमित हो सकता है। चूँकि रवैया नियंत्रक के पास अनंत बैंडविड्थ नहीं है, रवैया ट्रेकिंग त्रुटि स्थिति और रवैया गतिशीलता के बीच एक मजबूत नॉनलाइनियर युग्मन का परिचय देती है। समग्र प्रणाली की स्थिरता पर इसके प्रभाव का आमतौर पर वर्तमान साहित्य में विश्लेषण नहीं किया गया है। इस थीसिस में, नॉनसिंगुलर कमांड रवैया निष्कर्षण सुनिश्चित करने के लिए हाइपरबोलिक स्पष्टिफ़ा फ़ंक्शन का उपयोग करके एक संतृप्ति नियंत्रण योजना स्थिति नियंत्रण के लिए डिज़ाइन की गई है। रवैया ट्रेकिंग के दौरान रवैया विलक्षणता की समस्या को दूर करने के लिए प्रारंभिक स्थिति बाधा के साथ बैरियर लायपुनोव फ़ंक्शन (बीएलएफ) का उपयोग करके एक पिच बाधित रवैया नियंत्रण प्रस्तावित है।

एक और चिंता यह है कि संलग्न रोबोटिक आर्म/मैनिपुलेटर का उपयोग करके भौतिक वातावरण के साथ बातचीत करने पर क्वाड्रोटर का ट्रेकिंग नियंत्रण खराब हो जाता है। ऐसा जड़त्व मापदंडों में परिवर्तन के कारण होता है। यह कार्य पैरामीटर अनिश्चितताओं की उपस्थिति में मजबूत ट्रेकिंग प्रदर्शन स्थापित करने के लिए अनुकूल पदानुक्रमित रवैया नियंत्रण का परिचय देता है। कठोर स्थिरता विश्लेषण के साथ एक मल्टीस्टेज रचनात्मक प्रक्रिया, दो उपप्रणालियों के बीच नॉनलाइनियर युग्मन के प्रभाव पर विचार करते हुए, क्वाड्रोटर प्रणाली के नॉनलाइनियर पदानुक्रमित नियंत्रण डिजाइन के हर चरण में संश्लेषित की गई है।

इसके अलावा, मुख्य प्रभाव जो दिखाई देते हैं और क्यूएमएस के गतिशील व्यवहार को मानक क्वाड्रोटर से अलग बनाते हैं, वे हाथ की गति से उत्पन्न गतिशील प्रतिक्रिया बल और टॉर्क के साथ-साथ भौतिक वस्तु के साथ संपर्क बनाते समय प्रभाव बल हैं। चूंकि क्यूएमएस एक जटिल मल्टीबॉडी सिस्टम है, इसलिए काम का यह हिस्सा पहले से डिजाइन किए गए नॉनलाइनर पदानुक्रमित नियंत्रण का विस्तार करता है और इसके युग्मित गतिशील व्यवहार और पर्यावरण इंटरैक्शन पहलुओं पर विचार करते हुए स्थिति, दृष्टिकोण और हेरफेर उपप्रणालियों पर एक साथ लागू होता है। पिच बाधा रवैया नियंत्रण डिज़ाइन को विलक्षणता समस्या को संबोधित करने के लिए अपनाया जाता है जो अन्यथा क्यूएमएस की स्थिरता को गंभीर रूप से प्रभावित कर सकता है जबकि मैनिपुलेटर अंतःप्रभावकार प्रभाव टकराव के माध्यम से स्थिर या गतिशील वस्तु के साथ संपर्क बनाने की कोशिश कर रहा है।

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Abbreviations

AS	A symptotic S tability
BLF	B arrier L yapunov F unction
CF	C ushioned F oor
CLF	C ontrol L yapunov F unction
CoG	C enter of G ravity
CoM	C enter of M ass
DoF	D egrees of F reedom
DSC	D ynamic S urface C ontrol
EE	E nd E ffector
GAS	G lobal A symptotic S tability
iBLF	I ntegral B arrier L yapunov F unction
LIP	L inear I n P arameter
LPF	L ow P ass F ilter
MIMO	M ultiple I nput M ultiple O utput
MRAC	M odel R eference A daptive C ontrol
MSM	M ass-spring M anipulation S ystem
QMS	Q uadrotor M anipulation S ystem
SGES	S emi-globally E xponentially S table
SGUUB	S emi-globally U niformly U ltimately B ounded
UMS	U nderactuated M echanical S ystem
UUB	U niformly U ltimately B ounded

Symbols

s	scalar quantity s
$\mathbb{N}$	set of all natural numbers
$\mathbb{R}$	set of all real numbers
$\mathbb{R}^+$	set of nonnegative real numbers
$\mathbb{R}_{>0}$	set of positive real numbers
$\mathbb{R}^n$	real space of dimension n
$\mathbb{R}^{n \times n}$	real space of matrices of dimension $n \times n$
$\mathbb{R}_+^{n \times n}$	real space of positive definite matrices of dimension $n \times n$
$\mathbb{D}^{n \times n}$	real space of diagonal matrices of dimension $n \times n$
$\mathbb{D}_+^{n \times n}$	real space of positive definite diagonal matrices of dimension $n \times n$
$\mathbb{R}^{m \times n}$	real space of matrices of dimension $m \times n$
$\exists$	there exists
$\forall$	for all
$ v $	absolute value of scalar quantity v
v_i	i^{th} element of vector v
$\ v\ $	Euclidean norm of vector v
$\ v\ _1$	1-norm of vector v
$\ A\ _F$	Frobenius norm of matrix A
$\ A\ _\infty$	Induced infinity norm of matrix A
$I_{3 \times 3}$	Identity matrix of dimension 3×3
$a \rightarrow b$	a goes to b
$a \in b$	a belongs to b
$a \triangleq b$	a is defined as b