

SELF-ORGANIZATION AND SYNTHETIC GAUGE FIELD
FOR ULTRA-COLD BOSONS FROM ATOM-PHOTON
INTERACTION INSIDE A CAVITY

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Poornima Shakya: *Self-organization and synthetic gauge field for ultra-cold bosons from atom-photon interaction inside a cavity*, © July 2023

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FOR ULTRA-COLD BOSONS FROM ATOM-PHOTON
INTERACTION INSIDE A CAVITY

by

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CERTIFICATE

This is to certify that the thesis entitled, "**Self-organization and synthetic gauge field for ultra-cold bosons from atom-photon interaction inside a cavity**", submitted by **Ms. Poornima Shakya**, to the Department of Physics, Indian Institute of Technology Delhi, for the award of degree of **Doctor of Philosophy**, is a record of bonafide research work carried out by her under my supervision. She has fulfilled the requirements for the submission of the thesis, which to the best of our knowledge has reached the required standard. The results obtained in this thesis have not been submitted in part or full to any other University or Institute for the award of any degree or diploma.

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Dedicated to my loving family...

Abstract

We investigate the self-organization of a condensate of ultra-cold bosons trapped in a linear cavity illuminated by a two-pump configuration making different angles with the cavity axis. We demonstrate that our proposed pump-cavity configuration provides a smooth transition from a one-dimensional to a two-dimensional cavity-generated optical lattice potential that arises from the dipole interaction between the atoms and the cavity photons. The critical cavity-pump detuning required to transition from a normal phase to a superradiant phase is obtained using a Holstein-Primakoff approximation. Our calculations resemble a Dicke-like phase transition under the limiting case of equal and orthogonal pump angles. The superradiant phase sees a sudden rise of cavity photon number accompanied by simultaneous self-organization of the atoms in lattice supersolid phases, confirmed by the atomic density profiles plotted in the normal and the superradiant regime. We also obtain the extended Bose-Hubbard model from the microscopic Hamiltonian of the system, and the resulting nearest-neighbour, on-site, and long-range interaction parameters give us a qualitative understanding of the emergent self-organized superradiant phases. We also identify the underlying symmetries of the Hamiltonian, which are spontaneously broken across the superradiant phase transformation and study the effect of the dimensional crossovers on the collective excitation spectrum of the atomic and the photonic fluctuations.

We also explore another single atom-ring cavity system which can generate an atomic analogue of the snake state trajectories of electrons encountered in condensed matter experimental systems such as two-dimensional electron gas and graphene p-n junctions, to name a few. The atom interacts with the running wave modes of the ring cavity to give rise to a complex Rabi frequency that accounts for the appearance of geometric gauge potentials and the corresponding synthetic gauge fields. We examine the atom's motion in such spatially varying magnetic fields representing electronic snake states' atomic analogue. The real-time observation and non-demolition measurements of the atom dynamics are carried out through the output cavity photons, which depend on the atom's position inside the cavity. Tuning the atom-photon coupling strength, the external pumping strength, the initial velocities of the atom and the cavity backaction via atom-photon coupling allow us to change the properties of these atomic trajectories. These atomic trajectories can be used for the

loss-less transport of ultracold atoms to long distances, which can have a wide range of applications in quantum computing and atomtronics.

शोध सार

हम गुहा अक्ष के साथ अलग-अलग कोण बनाते हुए दो-पंप विन्यास द्वारा प्रकाशित एक रैखिक गुहा में फंसे अल्ट्रा-ठंडे बोसोन के संघनन के स्व-संगठन की जांच करते हैं। हम प्रदर्शित करते हैं कि हमारा प्रस्तावित पंप-गुहा विन्यास एक-आयामी से दो-आयामी गुहा-जनित ऑप्टिकल जाली क्षमता में एक सहज संक्रमण प्रदान करता है जो परमाणुओं और गुहा फोटॉन के बीच द्विध्रुवीय संपर्क से उत्पन्न होता है। सामान्य चरण से सुपररेडियंट चरण में संक्रमण के लिए आवश्यक महत्वपूर्ण कैविटी-पंप डिट्यूनिंग को होल्स्टीन-प्रिमाकोफ़ सन्निकटन का उपयोग करके प्राप्त किया जाता है। हमारी गणना समान और ऑर्थोगोनल पंप कोणों के सीमित मामले के तहत डिकी-जैसे चरण संक्रमण से मिलती जुलती है। सुपररेडियंट चरण में जाली सुपरसॉलिड चरणों में परमाणुओं के एक साथ स्व-संगठन के साथ गुहा फोटॉन संख्या में अचानक वृद्धि देखी जाती है, जिसकी पुष्टि सामान्य और सुपररेडियंट शासन में प्लॉट किए गए परमाणु घनत्व प्रोफाइल द्वारा की जाती है। हम सिस्टम के सूक्ष्म हैमिल्टनियन से विस्तारित बोस-हबर्ड मॉडल भी प्राप्त करते हैं, और परिणामी निकटतम-पड़ोसी, ऑन-साइट और लंबी दूरी के इंटरैक्शन पैरामीटर हमें उभरते स्व-संगठित सुपररेडियंट चरणों की गुणात्मक समझ देते हैं। हम हैमिल्टनियन की अंतर्निहित समरूपताओं की भी पहचान करते हैं, जो सुपररेडियंट चरण परिवर्तन में स्वचालित रूप से टूट जाती हैं और परमाणु और फोटोनिक उतार-चढ़ाव के सामूहिक उत्तेजना स्पेक्ट्रम पर आयामी क्रॉसओवर के प्रभाव का अध्ययन करते हैं।

हम एक अन्य एकल परमाणु-रिंग गुहा प्रणाली का भी पता लगाते हैं जो संघनित पदार्थ प्रायोगिक प्रणालियों जैसे कि दो-आयामी इलेक्ट्रॉन गैस और ग्राफीन पी-एन जंक्शनों में पाए जाने वाले इलेक्ट्रॉनों के साँप राज्य प्रक्षेपक का एक परमाणु एनालॉग उत्पन्न कर सकता है। परमाणु एक जटिल रबी आवृत्ति को जन्म देने के लिए रिंग गुहा के चलने वाले तरंग मोड के साथ बातचीत करता है जो ज्यामितीय गेज क्षमता और संबंधित सिंथेटिक गेज क्षेत्रों की उपस्थिति के लिए जिम्मेदार होता है। हम इलेक्ट्रॉनिक साँप राज्यों के परमाणु एनालॉग का प्रतिनिधित्व करने वाले ऐसे स्थानिक रूप से भिन्न चुंबकीय क्षेत्रों में परमाणु की गति की जांच करते हैं। परमाणु गतिशीलता का वास्तविक समय अवलोकन और गैर-विध्वंस माप आउटपुट कैविटी फोटॉन के माध्यम से किया जाता है, जो गुहा के अंदर परमाणु की स्थिति पर निर्भर करता है। परमाणु-फोटॉन युग्मन शक्ति, बाहरी पंपिंग शक्ति, परमाणु के प्रारंभिक वेग और परमाणु-फोटॉन युग्मन के माध्यम से गुहा बैकएक्शन को ट्यून करने से हमें इन परमाणु प्रक्षेप पथों के गुणों को बदलने की अनुमति मिलती है। इन परमाणु प्रक्षेप पथों का उपयोग लंबी दूरी तक अल्ट्राकोल्ड परमाणुओं के हानि-रहित परिवहन के लिए किया जा सकता है, जिसमें क्वांटम कंप्यूटिंग और एटमट्रॉनिक्स में अनुप्रयोगों की एक विस्तृत श्रृंखला हो सकती है।

PUBLICATIONS

1. **Poornima Shakya**, Amulya Ratnakar and Sankalpa Ghosh,
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2. **Poornima Shakya**, Nishant Dogra and Sankalpa Ghosh,
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3. **Poornima Shakya**, and Sankalpa Ghosh,
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*Turns out, not where, but who
you're with that really matters.*

— *Anonymous*

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