

FORMULATION OF GRAPH THEORETIC  
STRUCTURE-SENSITIVE  
 $Z_{Bus}$  MODELS FOR POWER SYSTEM ANALYSIS

by

D. ANAND RAO

ELECTRICAL ENGINEERING DEPARTMENT

submitted

in fulfilment of the requirements

of the degree of

DOCTOR OF PHILOSOPHY

to the

INDIAN INSTITUTE OF TECHNOLOGY, DELHI  
1983

## C E R T I F I C A T E

Certified that the research work "Formulation of Graph Theoretic Structure - Sensitive  $Z_{\text{Bus}}$  Models for Power System Analysis" by D. Anand Rao, has been carried out under our joint supervision at the Indian Institute of Technology, Delhi, and that this work has not been submitted elsewhere for the award of a Degree.

  
(J. Nanda) 30 8 83  
Professor

  
(P.S. Satsangi) 30/8/83  
Professor

Department of Electrical Engineering  
Indian Institute of Technology, Delhi  
New Delhi 110016

## ACKNOWLEDGEMENTS

The researcher wishes to express his deep and undying gratitude to, his supervisors, Professor Prem S. Satsangi and Professor J.Nanda for their able guidance, help and encouragement at every stage in completing this research presentation.

The researcher gratefully acknowledges the generous help extended by Madam, Director, Dr. Mrs. G.P. Sherry and Professor D.Amrit Swarup, Dean, Faculty of Engineering, Dayalbagh Educational Institute, Dayalbagh, Agra, and authorities of Indian Institute of Technology, Delhi.

Sincere thanks are due to my colleagues and friends, particularly Dr. B.P. Singh, Shri A.K. Tandon and Dr V.G.Das, who have contributed directly or indirectly to the success of this venture.

Excellent and skilful typing by Shri J.Anand Rao is sincerely appreciated.

Finally, I would like to thank my wife, Preeti, for her care and patience during the entire research schedule.

D. ANAND RAO

## A B S T R A C T

Power System Analysis usually entails the computation of system network voltages and currents under a given set of conditions. Under abnormal fault conditions, Short Circuit Studies determine the voltages and currents at the system buses. Bus Impedance Matrix,  $Z_{Bus}$ , is the preferred model for Short Circuit Analyses of Large Interconnected Power Systems using Digital Computers.

The main thrust of the present work is to explore and demonstrate the practical applicability and effectiveness of the Physical Systems Modelling Methodology based upon the Linear Graph Theory. Investigations made in the course of this thesis highlight the prudent application of a Physical Systems Modelling Construct in advantageously exploiting the structure of the system network for the purpose of the  $Z_{Bus}$  model and thereby, exhibiting features of orderliness and generality that go far beyond the mere manipulation of the Z-matrix driving point and transfer impedances.

The research presentation is organised in six chapters as given below.

Chapter-I presents the literature survey of existing techniques for short circuit studies and associated algorithms for  $Z_{Bus}$  modelling.

Chapter-II presents the Symbolic formulation pertaining to the application of the Graph Theoretic Chord Formulation Technique using a component-to-sub-system-to-system construct for obtaining the  $Z_{BUS}$  model at the  $(i+1)^{th}$  level of interconnection from a basis of the  $i^{th}$  level configuration. Simple relations in the form of equations have been derived for modifying the  $Z_{BUS}$  matrix corresponding to the addition of a tree branch or a chord of cotree. Ability of the graph theoretic approach - (i) to interconnect one or more elements at any desired level of interconnection and (ii) to identify and model appropriate sub-assemblies compositely in isolation - has been highlighted.

Chapter-III presents an algorithm for efficient modelling of mutually coupled lines. Representation of mutual coupling by appropriate current dependent voltage driver(s) is suggested. Mutually coupled groups of lines are identified as a distinct type of sub-assemblies and are modelled compositely in isolation. The sub-system representation of a coupled group of elements such that the mutual coupling effect is represented by appropriate network elements (current dependent voltage drivers) permits the modelling of the coupled group after the pattern of an uncoupled sub-system which, any way, can be easily modelled according to relevant formulae developed in Chapter-II. Computationally efficient methods have been proposed for simulating minor changes in the structure of the system.

Chapter-IV identifies two distinctive structural features of the system network configuration which have been considered for effective and efficient modelling. One aspect relates to the identification of a cantilever type of sub-structure and exploitation of its associated attributes. The other aspect relates to the optimal structuring and ordering of f-circuits for an efficient formulation of the  $Z_{\text{Bus}}$  model. Exploitation of these structural features of the system network configuration has resulted in substantial reduction of computer processing time which is highlighted in terms of reduction in arithmetic operations required for the  $Z_{\text{Bus}}$  formation.

Chapter-V presents algorithms for - (i) developing an inter area  $Z_{\text{Bus}}$  model corresponding to interconnection of two or more areas to power grids through tie-lines and (ii) identifying study sub-systems in large power systems from a basis of Technical and rational criteria for such sub-divisions in the absence of apriori specifications of study system boundaries.

Chapter-VI presents the overall conclusions arising from the research investigations carried out in this thesis and suggests further avenues of research leading therefrom.

## TABLE OF CONTENTS

|                                                                                                                               |     |      |
|-------------------------------------------------------------------------------------------------------------------------------|-----|------|
| Abstract                                                                                                                      | ..  | viii |
| List of Principal Symbols<br>and Abbreviations.                                                                               | ..  | xi   |
| CHAPTER-I INTRODUCTION                                                                                                        |     |      |
| 1.1 Models for Power System Analysis                                                                                          | ..  | 1    |
| 1.2 Brief History of $Z_{Bus}$ Formulation<br>for Short Circuit Studies.                                                      | ..  | 4    |
| 1.2.1 Development of $Z_{Bus}$ Model<br>for Fault Analysis.                                                                   | ..  | 4    |
| 1.2.2 Methods of Short Circuit<br>Analysis.                                                                                   | ..  | 7    |
| 1.2.3 Methods of $Z_{Bus}$ Modelling                                                                                          | ..  | 11   |
| 1.3 Scope And Outline of the Thesis                                                                                           | ... | 19   |
| CHAPTER-II GRAPH THEORETIC CHORD FORMULATION<br>TECHNIQUE FOR DEVELOPING $Z_{Bus}$ MODEL<br>IN THE ABSENCE OF MUTUAL COUPLING |     |      |
| 2.1 Introduction                                                                                                              | ..  | 28   |
| 2.2 Large Scale Systems view-point<br>of Power System Modelling Via<br>Linear Graph Theory.                                   | ..  | 30   |
| 2.2.1 Systems View Point of<br>$Z_{Bus}$ Modelling.                                                                           | ..  | 30   |
| 2.2.2 Linear Graph Theory In<br>Systems Perspective.                                                                          | ..  | 32   |
| 2.2.3 Formulation Techniques for<br>Analytical Solution.                                                                      | ..  | 35   |
| 2.3 Component-to-sub-systems-to-<br>system Construct based upon<br>Graph Theoretic Chord Formulation<br>Technique.            | ..  | 37   |

(ii)

|       |                                                                                                                        |    |    |
|-------|------------------------------------------------------------------------------------------------------------------------|----|----|
| 2.3.1 | Sub-system Representation at the $(i+1)^{th}$ level.                                                                   | .. | 38 |
| 2.3.2 | Symbolic Formulation                                                                                                   | .. | 42 |
| 2.4   | B-Sub-matrix Corresponding to sub-system representation, $S_i$ , at the $i^{th}$ level of Interconnection              | .. | 52 |
| 2.4.1 | Fundamental Circuit Equations.                                                                                         | .. | 53 |
| 2.4.2 | B-Matrix at the $i^{th}$ level of interconnection.                                                                     | .. | 55 |
| 2.5   | Formation of B-sub-matrix when a Component $C_{i+1}$ is added at the $(i+1)^{th}$ level of Interconnection.            | .. | 60 |
| 2.5.1 | Formation of B-sub-matrix when $C_{i+1}$ is a tree branch of $F_{i+1}$                                                 | .. | 61 |
| 2.5.2 | Formation of B-sub-matrix when $C_{i+1}$ is a chord of the cotree of $F_{i+1}$                                         | .. | 64 |
| 2.5.3 | Rule for formation of $B_{i+1}$ at the $(i+1)^{th}$ level.                                                             | .. | 66 |
| 2.6   | Formation of $Z_{Bus}$ at the $(i+1)^{th}$ level of interconnection.                                                   | .. | 67 |
| 2.6.1 | Formation of the Elements of the $(i+1)^{th}$ row and column of $Z_{i+1}$ when $C_{i+1}$ is a tree branch of $F_{i+1}$ | .. | 69 |
| 2.6.2 | Formation of the elements of $Z_{Bus}$ when $C_{i+1}$ is added as a chord of the cotree of $F_{i+1}$                   | .. | 72 |



(iii)

|                                                                                                                    |                                                                      |    |     |
|--------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------|----|-----|
| 2.7                                                                                                                | Modification of $Z_{Bus}$ for minor changes in the network.          | .. | 75  |
| 2.7.1                                                                                                              | Addition of a new line                                               | .. | 75  |
| 2.7.2                                                                                                              | Removal or opening of a line.                                        | .. | 76  |
| 2.7.2.1                                                                                                            | Removal of line pq without eliminating either Bus p or q.            | .. | 79  |
| 2.7.2.2                                                                                                            | Removal of line pq which reduces the number of buses by one.         | .. | 86  |
| 2.7.3                                                                                                              | Change of line impedance                                             | .. | 87  |
| 2.8                                                                                                                | Sample Example                                                       | .. | 88  |
| 2.8.1                                                                                                              | Formation of $Z_{Bus}$                                               | .. | 88  |
| 2.8.2                                                                                                              | Removal of a line                                                    | .. | 91  |
| 2.8.3                                                                                                              | Change of Impedance of a line.                                       | .. | 92  |
| 2.9                                                                                                                | Conclusions                                                          | .. | 93  |
| CHAPTER-III GRAPH THEORETIC CHORD FORMULATION TECHNIQUE FOR DEVELOPING $Z_{Bus}$ MODEL WITH MUTUALLY COUPLED LINES |                                                                      |    |     |
| 3.1                                                                                                                | Introduction                                                         | .. | 97  |
| 3.2                                                                                                                | Formation of Mutually Coupled Groups.                                | .. | 100 |
| 3.2.1                                                                                                              | Coupled Group of Elements                                            | .. | 101 |
| 3.2.2                                                                                                              | Formation of Coupled Groups                                          |    | 102 |
| 3.3                                                                                                                | Representation of Mutual Coupling                                    | .. | 105 |
| 3.4                                                                                                                | Systems Approach for Efficient Modelling of Mutually Coupled Groups. | .. | 108 |

(iv)

|       |                                                                                                                                                |    |     |
|-------|------------------------------------------------------------------------------------------------------------------------------------------------|----|-----|
| 3.4.1 | Methodology of modelling coupled lines.                                                                                                        | .. | 108 |
| 3.4.2 | Change of reference bus                                                                                                                        | .. | 112 |
| 3.4.3 | Interconnection of the $Z_{\text{Bus}}$ Model of a coupled group to the system.                                                                | .. | 116 |
| 3.5   | Formation of $Z_{\text{Bus}}$ of a coupled group.                                                                                              | .. | 124 |
| 3.5.1 | Modification of $Z_{\text{Bus}}$ corresponding to the addition of a current dependent voltage driver at the $(i+1)^{\text{th}}$ level.         | .. | 124 |
| 3.5.2 | Modification of $Z_{\text{Bus}}$ corresponding to the addition of a tree branch with mutual coupling at the $(i+1)^{\text{th}}$ level.         | .. | 132 |
| 3.5.3 | Modification of $Z_{\text{Bus}}$ corresponding to the addition of a chord of the cotree with mutual coupling at the $(i+1)^{\text{th}}$ level. | .. | 137 |
| 3.5.4 | Stepwise procedure for obtaining $Z_{\text{Bus}}$ model of a coupled group of elements.                                                        | .. | 143 |
| 3.6   | Modification of $Z_{\text{Bus}}$ for minor changes in the system involving mutually coupled lines.                                             | .. | 146 |
| 3.6.1 | Method 1                                                                                                                                       | .. | 147 |
| 3.6.2 | Method 2                                                                                                                                       | .. | 152 |
| 3.7   | Sample Example                                                                                                                                 | .. | 156 |
| 3.7.1 | Formation of $Z_{\text{Bus}}$ by new graph theoretic approach                                                                                  | .. | 158 |

( v )

|       |                                                                                  |    |     |
|-------|----------------------------------------------------------------------------------|----|-----|
| 3.7.2 | $Z_{\text{Bus}}$ formation by the old $Z_{\text{Bus}}$ algorithm.                | .. | 168 |
| 3.7.3 | Removal of a coupled element by the graph theoretic approach.                    | .. | 176 |
| 3.7.4 | Removal of a coupled element by the old $Z_{\text{Bus}}$ algorithm.              | .. | 178 |
| 3.7.5 | Selective Row Generation for obtaining $Y_{\text{PQ-PQ}}$ and $Y_{\text{PQ-RS}}$ | .. | 180 |
| 3.7.6 | Addition of a mutually coupled line.                                             | .. | 187 |
| 3.7.7 | Change in Primitive Impedance of a coupled line                                  | .. | 189 |
| 3.8   | Conclusions                                                                      | .. | 191 |

CHAPTER-IV. EFFICIENT FORMATION OF  $Z_{\text{Bus}}$  THROUGH STRUCTURE-SENSITIVE SYSTEMS APPROACH USING GRAPH THEORETIC CHORD FORMULATION TECHNIQUE.

|       |                                                                                                                        |    |     |
|-------|------------------------------------------------------------------------------------------------------------------------|----|-----|
| 4.1   | Introduction                                                                                                           | .. | 192 |
| 4.2   | Old Approaches for $Z_{\text{Bus}}$ formation and their limitations.                                                   | .. | 193 |
| 4.3   | Cantilever sub-systems                                                                                                 | .. | 195 |
| 4.3.1 | $Z_{\text{Bus}}$ matrix                                                                                                | .. | 195 |
| 4.3.2 | Addition of a cantilever branch at the $(i+1)^{\text{th}}$ level of interconnection at bus p.                          | .. | 197 |
| 4.3.3 | Addition of a cantilever sub-system.                                                                                   | .. | 205 |
| 4.3.4 | Derivation of the elements of $Z_{\text{Bus}}$ when a cantilever sub-system is added at the $(i+1)^{\text{th}}$ level. | .. | 212 |

|                                                                                                                            |                                                                                                          |    |     |
|----------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------|----|-----|
| 4.4                                                                                                                        | Appropriate level of interconnection for chords of the system cotree.                                    | .. | 218 |
| 4.5                                                                                                                        | Sample Example                                                                                           | .. | 229 |
| 4.5.1                                                                                                                      | Stepwise $Z_{Bus}$ formation of the sample system through Brown's algorithm.                             | .. | 231 |
| 4.5.2                                                                                                                      | Stepwise $Z_{Bus}$ formation of the sample system through the systemic approach.                         | .. | 249 |
| 4.6                                                                                                                        | Comparison of the old and new algorithms.                                                                | .. | 265 |
| 4.7                                                                                                                        | Conclusions                                                                                              | .. | 265 |
| CHAPTER-V FORMATION OF MULTI-AREA $Z_{Bus}$ AND IDENTIFICATION OF STUDY SUB-SYSTEMS IN A LARGE INTERCONNECTED POWER SYSTEM |                                                                                                          |    |     |
| 5.1                                                                                                                        | Introduction                                                                                             | .. | 267 |
| 5.2                                                                                                                        | Formation of multi-area $Z_{Bus}$ using graph theoretic chord formulation technique.                     | .. | 271 |
| 5.3                                                                                                                        | Numerical Example                                                                                        | .. | 276 |
| 5.4                                                                                                                        | Hierarchical grouping of related elements based upon strength of connectivity.                           | .. | 282 |
| 5.4.1                                                                                                                      | Numerical Example                                                                                        | .. | 283 |
| 5.5                                                                                                                        | Hierarchical decomposition of a set which has an associated Linear graph-based upon flow of information. | .. | 288 |
| 5.5.1                                                                                                                      | Numerical example.                                                                                       | .. | 289 |
| 5.6                                                                                                                        | Conclusions                                                                                              | .. | 291 |

CHAPTER-VI CONCLUSIONS

|     |                                                                                                                             |    |     |
|-----|-----------------------------------------------------------------------------------------------------------------------------|----|-----|
| 6.1 | Introduction                                                                                                                | .. | 294 |
| 6.2 | Application of the chord<br>formulation technique by using a<br>component-to-sub-system-to-system<br>construct.             | .. | 294 |
| 6.3 | $Z_{Bus}$ formation of mutually<br>coupled lines.                                                                           | .. | 295 |
| 6.4 | Systemic structure exploitation                                                                                             | .. | 297 |
| 6.5 | Formation of the combined $Z_{Bus}$<br>model of Interconnected power<br>systems and identification of study<br>sub-systems. | .. | 298 |
| 6.6 | Scope for future research                                                                                                   | .. | 300 |

|            |    |     |
|------------|----|-----|
| REFERENCES | .. | 302 |
|------------|----|-----|

APPENDICES

|            |    |     |
|------------|----|-----|
| Appendix-A | .. | 313 |
| Appendix-B | .. | 317 |
| Appendix-C | .. | 321 |
| Appendix-D | .. | 331 |
| Appendix-E | .. | 338 |
| Appendix-F | .. | 353 |
| Appendix-G | .. | 357 |

|                  |    |     |
|------------------|----|-----|
| CURRICULUM VITAE | .. | 361 |
|------------------|----|-----|