Parallel Delaunay Triangulation for Finite Element Methods

by

Siba Charan Pradhan

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TO

MY PARENTS

AND

TEACHERS

CERTIFICATE

This is to certify that the thesis entitled

Parallel Delaunay Triangulation for Finite Element Methods

which is being submitted by **Siba Charan Pradhan**, Research Scholar, Department of Mathematics, to the Indian Institute of Technology, Delhi for the award of the degree of Doctor of Philosophy in Mathematics, is a record of bonafide research work carried out by him under our supervision and has fulfilled all the requirements for the submission of this thesis.

The results contained in this thesis have not been submitted in part or full, to any other university or institute for the award of any other degree or diploma.

P. K. Bhattacharyya

(P. K. BHATTACHARYYA)
Professor(Retd.),
Department of Mathematics,
Indian Institute of Technology,
DELHI.

14.07.99

S. R. K. Iyengar

(S. R. K. IYENGAR)
Professor,
Department of Mathematics,
Indian Institute of Technology,
DELHI.

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Siba Charan Pradhan

ABSTRACT

The present thesis deals with the problem of automatic triangulation (also known as mesh generation) for finite element methods under a parallel environment, consisting of a network of several workstations, which are easily available. The present thesis addresses itself

(1) to develop a suitable data structure which efficiently manages various traversal / search operations during different algorithmic steps of Delaunay triangulation with a special emphasis to accommodate several Delaunay triangulation algorithms like incremental point insertion scheme and local transformation;

(2) to implement these algorithms for Delaunay triangulation under this data structure;

(3) to experiment with various interior point generation schemes and to develop a scheme suitable for parallel mesh generation;

(4) to present a practical implementation of parallel Delaunay triangulation algorithm and parallel Laplace Delaunay smoothing under a set of networked workstations, which is cheap and is widely available.

This thesis has been organized into six chapters.

Chapter 1 contains the motivation of the research problem undertaken in this thesis and the survey of the existing research literature on unstructured mesh generation schemes using Delaunay Triangulation/ Voronoi Diagram along with its parallel implementations [28, 29, 53, 147] and also the present status of this research problem.

Chapter 2 describes the sequential algorithm under an associative data structure, which is the building block for parallel Delaunay triangulation. The details of the data structure have been discussed. This data structure organizes all mesh topological entities and geometrical entities under a hierarchical framework. Associativity among each entity in each hierarchy is maintained so as to make traversals and search operations during Delaunay Triangulation efficient. A brief description of the incremental point insertion algorithm [19, 138] for Delaunay Triangulation along with its computational components like target oriented geometric search and tree search to find the final Delaunay cavity, has been presented along with results of numerical experiments showing the efficiency of these computational components under the present data structure.

Chapter 3 is devoted to the automatic point generation scheme for the interior of the domain, which will be suitable for parallel processing. It describes certain innovative improvements in the incremental circumcenter insertion scheme for automatic interior point generation originally presented by Synder [52] so that the resulting mesh becomes more suitable for parallel processing. The basic requirement for parallel processing is the divide-and-conquer strategy, which enforces that sub-domain triangulations are such that the union of the sub-domain triangulations should give rise to the global Delaunay triangulation. Hence, it is required to generate interior points in each sub-domain such that the triangulation obtained is either globally Delaunay or needs very little local transformation to maintain globally Delaunay property for the entire triangulation of the domain. Detailed explanation is presented to the case when a particular order is maintained, while generating interior points within each sub-domain. This order in conjunction with the constraints applied to the interfaces among sub-domains, presented in Chapter 4, minimizes the need for local transformation after gluing sub-domain meshes obtained from several slave processors.

Chapter 4 presents certain prerequisites such as a unique point numbering scheme and a problem decomposition strategy to be taken into consideration before proceeding with the parallel mesh generation. The unique point numbering scheme basically obviates the need for point renumbering after getting triangulations from slave processors and joining them to obtain final triangulation. Other issues like the choice of a parallel architecture and problem decomposition strategy have been discussed. The problem of mesh generation has been decomposed functionally into 3 components viz. (i) geometry input function, (ii) triangulation function, (iii) result presentation and visualization. Then, a data decomposition strategy has been applied to the triangulation function in order to apply the triangulation algorithm to these decomposed data subsets in parallel. These data subsets in the current context are geometric sub-domains of the original domain given for triangulation. Certain constraints on the (i) interfaces between adjacent sub-domains and (ii) universal angle criteria necessary for finite element applications are maintained as far as possible during partitioning of the original domain. These constraints in conjunction with the automatic point generation scheme described in Chapter 3 minimize the computational requirement during the post-processing step.

Chapter 5 is devoted to the actual parallel implementation strategy. The parallel environment in the present case is a set of workstations connected through a shared bus topology (called Ethernet LAN) using standard ethernet communication protocol called CSMA-CD (Carrier Sense Multiple Access – Collision Detect). To utilize the shared bus topology in the best possible way, a master-slave communication strategy has been used along with dynamic load balancing for task distribution among the available slave processors. Under this arrangement geometry input, task distribution, and result visualization functions are given to the front-end workstation having specialized hardware for processing graphics data. These functions run as independent tasks. This front-end workstation is the master controller and is responsible for (i) finding other available processors, (ii) spawning triangulation programs in them, (iii) distributing sub-domain geometry into these spawned tasks, and (iv) receiving the results from these slave processors. Other workstations are used as compute nodes doing only triangulation job supplied to them by the master controller. Results of numerical experiments are presented for different processor configuration. Parallel Virtual Machine (PVM) [33, 41, 171] has been utilized to take care of the process control requirements and data communication needs among all the processors.

Chapter 6 describes post-processing requirements in parallel environment. Post-processing consists of two phases. The first phase is the smoothing [34, 38, 39] process in parallel environment. The present approach is a variant of the Laplace-Delaunay [34] smoothing process applied to all the interior nodes in each sub-domain in parallel environment. According to this approach a node is moved to its new position determined by the Laplace method only if it satisfies the Delaunay criteria and it improves the "quality" of the mesh locally. The term "quality" depends upon the specific application. For our application to the finite element methods we have applied additional criteria like (i) improvement to the minimum angle, (ii) improvement to the aspect ratios of the triangles etc. while validating the points created by the Laplace method before accepting them as suitable nodes in the triangulation. This is done within the parallel loop itself. Then, the results obtained from the slave processors are glued together to obtain a triangulation for the entire domain. The triangulation obtained at this stage needs further processing because of the fact that the interface edges may not satisfy the Delaunay criteria after gluing the sub-domain triangulations obtained from several slave processors, even though the resulting triangulation may be suitable for the finite element methods. This, in turn, will make the triangulation invalid for the purpose of further application of refinement/ de-refinement/ smoothing procedures based on the Delaunay criteria. The second phase of post-processing has been designed to circumvent this problem. This phase consists of recursive local transformation [58, 59] to interface edges in order to make entire triangulation Delaunay conforming followed by smoothing of the interface nodes using the same method as in the first phase. This step at this stage is a sequential process applied after parallel step is over.

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