ALGORITHMIC ASPECTS OF LIAR'S DOMINATION
AND
ITS VARIATIONS

by

SUBHABRATA PAUL

Department of Mathematics

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Dedicated to

My Family

Certificate

This is to certify that the thesis entitled **“Algorithmic Aspects of Liar’s Domination and Its Variations”** submitted by **“Mr. Subhabrata Paul”** to the Indian Institute of Technology Delhi, for the award of the Degree of Doctor of Philosophy, is a record of the original bona fide research work carried out by him under my supervision and guidance. The thesis has reached the standards fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted in part or full to any other university or institute for the award of any degree or diploma.

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Dr. Bhawani Sankar Panda
Professor
Department of Mathematics
IIT Delhi

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Abstract

A subset $L \subseteq V$ of a graph $G = (V, E)$ is called a *liar's dominating set* of G if (i) $|N_G[u] \cap L| \geq 2$ for every vertex $u \in V$ and (ii) $|(N_G[u] \cup N_G[v]) \cap L| \geq 3$ for every pair of distinct vertices $u, v \in V$. The *liar's domination number*, denoted by $\gamma_{LR}(G)$, is the minimum cardinality of a liar's dominating set of a graph G. The Minimum Liar's Domination Problem is to find a liar's dominating set of minimum cardinality of a given graph G. Given a graph G and a positive integer k, the Liar's Domination Decision Problem is to check whether G has a liar's dominating set of cardinality at most k.

P. J. Slater [Networks, 54(2), 2009, 70–74] has proved that the Liar's Domination Decision Problem is NP-complete for general graphs. Later, M. L. Roden and P. J. Slater [Discrete Mathematics, 309(19), 2009, 5884–5890] have improved this result by showing that this problem remains NP-complete for bipartite graphs.

Motivated by the fact that Liar's Domination Decision Problem is NP-complete for general graphs, in this thesis, we have studied liar's domination problem in some special graph classes.

We have proposed a linear time algorithm for finding a minimum liar's dominating set in a tree. We have also characterized the set of all vertices that belongs to every minimum liar's dominating of a tree and the set of all vertices that does not belong to any minimum liar's dominating set of a tree. Based on this characterization, we have proposed a polynomial time algorithm for finding these sets.

We have proposed linear time algorithms to find a minimum liar's dominating sets in block graphs and proper interval graphs. We have strengthened the NP-completeness result of LIAR DOMINATION DECISION PROBLEM by showing that this problem remains NP-complete for split graphs, undirected path graphs, and doubly chordal graphs; three important subclasses of chordal graphs.

Since MINIMUM LIAR'S DOMINATION PROBLEM is NP-hard for general graphs, we have also studied approximation algorithm for this problem. We have proposed an approximation algorithm and also obtained approximation hardness results for general graphs. We have shown that that MINIMUM LIAR'S DOMINATION PROBLEM is APX-complete for graphs with maximum degree 4. We have also shown that LIAR'S DOMINATION DECISION PROBLEM remains NP-complete for p-claw free graphs and obtained some hardness and approximation results of this problem for p-claw free graphs.

In this thesis, we have initiated the study of two new variations of liar's dominating set, namely *total liar's dominating set* and *connected liar's dominating set*. We have given a necessary and sufficient condition for the existence of a total liar's dominating set in a graph. We have proposed a linear time algorithm for minimum connected liar's domination problem for block graphs. We have shown that the decision version of these two problems, that is, TOTAL LIAR'S DOMINATION DECISION PROBLEM and CONNECTED LIAR'S DOMINATION DECISION PROBLEM are NP-complete for split graphs, undirected path graphs, and doubly chordal graphs. We have also obtain some approximation hardness and approximation algorithms for both of these problems for general graphs. Finally we have shown that MINIMUM TOTAL LIAR'S DOMINATION PROBLEM and MINIMUM CONNECTED LIAR'S DOMINATION PROBLEM are APX-complete for graphs with maximum degree 4.

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