

**INTEGRATED STRUCTURAL HEALTH  
MONITORING AND ENERGY HARVESTING  
POTENTIAL OF ADHESIVELY BONDED THIN PIEZO  
PATCHES OPERATING IN  $d_{31}$  MODE**

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**DEPARTMENT OF CIVIL ENGINEERING  
INDIAN INSTITUTE OF TECHNOLOGY DELHI**

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by

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**Submitted**

**in fulfilment of the requirements of the degree of**

**Doctor of Philosophy**

to the



**INDIAN INSTITUTE OF TECHNOLOGY DELHI**

**MAY 2015**

*Dedicated to my family and teachers  
for their support and guidance.*



## **CERTIFICATE**

This is to certify that the thesis titled “**INTEGRATED STRUCTURAL HEALTH MONITORING AND ENERGY HARVESTING POTENTIAL OF ADHESIVELY BONDED THIN PIEZO PATCHES OPERATING IN  $d_{31}$  MODE**” which is being submitted by **Ms. NAVEET KAUR** for the fulfilment of the requirements for the award of degree of **Doctor of Philosophy**, is a record of the student’s own work carried out at the **Indian Institute of Technology Delhi** under my supervision and guidance. The matter embodied in this thesis has not been submitted elsewhere for the award of any other degree or diploma.

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## **ABSTRACT**

Piezoelectric materials have so far proven their efficacy for both energy harvesting and structural health monitoring (SHM) individually. Piezoelectric ceramic (PZT) patches, operating in  $d_{31}$ -mode, are considered best for SHM. However, for energy harvesting, built up configurations such as stack actuators are more preferred. The main objective of this research was to explore the possibility of employing the same piezo sensor for SHM as well as energy harvesting on real-life civil structures such as bridges. Experiments have been carried out in the laboratory environment to measure the voltage and the power generated by PZT patches in surface bonded as well as embedded  $d_{31}$  configurations. In embedded form, the PZT patches have been considered in the form of concrete vibration sensor (CVS), operating in the normal axial strain ( $d_{31}$ ) mode. This packaged sensor (CVS), which is composite in nature, has better compatibility with the surrounding concrete and can withstand the harsh conditions typically encountered during casting. Analytical models have been developed based on Euler Bernoulli's theory for a beam under harmonic vibrations, using which the voltage output of a PZT patch, either surface bonded or embedded (CVS), can be determined duly considering the effects of shear lag and other losses associated with the bonding layers. Beam structure has been considered for modelling as well as lab experimentation owing to the energy harvesting potential offered by real-life bridges. Utilization of the same patch for energy harvesting as well as for SHM through a combination of the global vibration and the local EMI techniques has been experimentally demonstrated. The strength gain and the fatigue characteristics of the real-life sized RC beam have been duly investigated via the embedded CVS in laboratory. The experimental observations have been found to match with theoretical predictions. The performance of the CVS is compared with surface-bonded PZT patch. The typical

measured output power from a piezo sensor  $10 \times 10 \times 0.3$  mm has been determined to be in microwatt range.

Optimum parameters of PZT patches have been arrived at by performing detailed numerical investigations. A comprehensive parametric study to investigate the effect of various parameters such as PZT geometric parameters, adhesive layer thickness and bond stiffness, which is otherwise difficult to perform experimentally, has been done. A numerical model has been developed for a real-life sized simply supported beam instrumented with (a) surface bonded PZT sensor (SBPS), and (b) embedded PZT patch in form of concrete vibration sensor (CVS), and coupled field analysis has been performed for the two configurations through finite element method (FEM). The results have been compared with the existing analytical model as well as the experimental data and the comparison has been found satisfactory. Effect of varying load resistance across the PZT patch on the close circuit power generated by the patch has also been investigated. The investigations show that a piezo thickness lying between 0.3 mm and 1 mm ensures maximum yield for SBPS. For CVS, the thickness above 1 mm is somewhat more beneficial. In addition, covering the SBPS with a layer of epoxy is also beneficial.

A real-life flyover with span of length 25 m has been numerically modelled to estimate the dynamic strain levels and hence, the power that could be produced in real-life structures. The numerical study predicts a yield of  $2.2 \mu\text{W}$  from a typical city flyover. This aids to quantify the gap between the voltage generated by the PZT patches under specific conditions in lab environment and corresponding to the strain levels produced in real-life structures. Harvesting potential of the structural vibration energy by PZT sensors during idle time is experimentally demonstrated and this concept has been extended to

eight real-life bridges across the world, based on the validated analytical model. Effect of losses such as mechanical loss, dielectric loss and shear lag loss, on the real-life structure have been duly considered. Computations show that the power yield could range anywhere from less than a microwatt (RC bridges) to over 25  $\mu\text{W}$  (steel bridges). These figures correspond to power densities over 30  $\mu\text{W}/\text{cm}^{-3}$  and 800  $\mu\text{W}/\text{cm}^{-3}$ , respectively. The study reveals that the PZT patches could harvest enough energy in reasonable time in the idle state so as to enable intermittent operation of SHM system. Typically, it could take only a couple of seconds to scavenge enough energy to power commercially available A/D converters.

Proof-of-concept demonstration of normal plate type piezo in axial strain mode in place of commercial expensive transducers for energy harvesting from real-life civil structure has been the original aspect of this study. It is expected that the outcomes of this research will pave way for dual use of the ordinary piezo patch in SHM as well as energy harvesting.

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## **LIST OF SYMBOLS**

|               |                                                                                         |
|---------------|-----------------------------------------------------------------------------------------|
| $a$           | Amplitude of vibration of beam                                                          |
| $A_{st}$      | Area of tensile reinforcement                                                           |
| $b$           | Width of RC beam                                                                        |
| $B$           | Susceptance                                                                             |
| $c$           | Equivalent damping                                                                      |
| $(C_F)_{exp}$ | Experimental calibration constant for piezo force-sensor                                |
| $(C_F)_{th}$  | Theoretical calibration constant for piezo force-sensor                                 |
| $C$           | Capacitance of piezo sensor                                                             |
| $C_a$         | Calibration constant for accelerometer                                                  |
| $C_b$         | Curvature of beam                                                                       |
| $C_n$         | Generalized damping                                                                     |
| $d$           | Depth of neutral axis                                                                   |
| $d'$          | Distance of the centre line of PZT patch from neutral axis of beam                      |
| $[d]$         | Third order piezoelectric strain coefficient tensor                                     |
| $D$           | Overall depth of beam                                                                   |
| $D'$          | Equivalent overall depth of beam( $=D-2t_c-2t_s-h$ )                                    |
| $D_3$         | Charge density                                                                          |
| $[D_e]$       | Electric displacement vector                                                            |
| $E$           | Young's modulus of elasticity                                                           |
| $E_3$         | Electric field along direction 3                                                        |
| $EI$          | Flexural rigidity of beam                                                               |
| $EI_g$        | Flexural rigidity of girder of flyover                                                  |
| $f$           | Operating frequency of dynamic shaker                                                   |
| $f_a, f_b$    | Forcing frequencies at which the amplitude is $1/\sqrt{2}$ times the resonant amplitude |
| $f_{br}$      | Natural frequency of bridge                                                             |
| $f_{ck}$      | Characteristic strength of concrete                                                     |
| $f_n$         | Resonance frequency                                                                     |
| $f_{sh}$      | Natural frequency of vibration of inertial-type shaker                                  |
| $f_y$         | Characteristic strength of reinforcement in the RC beam                                 |
| $F$           | Force                                                                                   |
| $g$           | Acceleration due to gravity                                                             |
| $G$           | Conductance                                                                             |
| $G_k^u$       | Undamaged conductance value                                                             |
| $G_k^d$       | Damaged conductance value                                                               |
| $G_s$         | Shear modulus of elasticity of bonding layer                                            |

|                                       |                                                                                                                  |
|---------------------------------------|------------------------------------------------------------------------------------------------------------------|
| $h$                                   | Thickness of PZT patch                                                                                           |
| $i$                                   | Instantaneous current flowing in power measuring circuit                                                         |
| $i_L$                                 | Current flowing in circuit with load resistance across PZT patch                                                 |
| $i_{\text{peak}}$                     | Maximum current flowing in power measuring circuit                                                               |
| $i_{\text{RMS}}$                      | RMS current flowing in power measuring circuit                                                                   |
| $I$                                   | Second moment of inertia of beam                                                                                 |
| $k$                                   | Equivalent stiffness                                                                                             |
| $k_{31}$                              | Coupling coefficient of the PZT patch                                                                            |
| $(K_b)_S, (K_b)_E$                    | Correction factor for the shear lag effect in bond layer for surface bonded and embedded PZT patch, respectively |
| $K_n$                                 | Generalized stiffness                                                                                            |
| $K_p$                                 | Correction factor due to Poisson's effect                                                                        |
| $(l_{\text{eff}})_E$                  | Effective length of embedded PZT patch                                                                           |
| $L$                                   | Length of beam                                                                                                   |
| $L_p$                                 | Length of PZT patch                                                                                              |
| $\bar{m}$                             | Mass per unit length of beam                                                                                     |
| $m_{P1}, m_{P2},$<br>$m_{P3}, m_{P4}$ | Mass of additional four mass plates in inertial shaker                                                           |
| $m_t$                                 | Mass of top plate in inertial shaker                                                                             |
| $M$                                   | Bending moment at any section of the beam                                                                        |
| $M_n$                                 | Generalized mass                                                                                                 |
| $M_u$                                 | Ultimate load carrying capacity of RC beam                                                                       |
| $n$                                   | Number of modes                                                                                                  |
| $p(x,t)$                              | Forcing function                                                                                                 |
| $p_o$                                 | Amplitude of dynamic load                                                                                        |
| $P$                                   | Instantaneous power generated by PZT patch                                                                       |
| $P_{\text{avg}}$                      | Average power generated by PZT patch                                                                             |
| $P_n$                                 | Generalized force                                                                                                |
| $P_{\text{peak}}$                     | Maximum power generated by PZT patch                                                                             |
| $P_{\text{RMS}}$                      | RMS power generated by PZT patch                                                                                 |
| $Q$                                   | Total charge across PZT patch                                                                                    |
| $q_n$                                 | Generalized displacement                                                                                         |
| $R_1, R_2$                            | Resistance used in power measuring circuit                                                                       |
| $R$                                   | Resistances measured by AD5933                                                                                   |
| $R_D$                                 | Dynamic magnification factor                                                                                     |
| $R_L$                                 | Load resistance across PZT patch                                                                                 |
| $[S]$                                 | Second order strain tensor                                                                                       |
| $(s_1)_S, (s_1)_E$                    | Strain in the topmost fibre of beam with surface-bonded and embedded PZT patch, respectively                     |

|                            |                                                                                                       |
|----------------------------|-------------------------------------------------------------------------------------------------------|
| $S_2$                      | Strain in the bottom-most fiber of the beam                                                           |
| $s_{11}^E$                 | Elastic compliance of PZT patch corresponding to 11 axis                                              |
| $S_b$                      | Strain in RC beam                                                                                     |
| $S_p$                      | Strain in PZT patch                                                                                   |
| $S_q^*$                    | Circuit sensitivity                                                                                   |
| $t_c$                      | Depth of PZT patch in embedded CVS from surface of concrete beam                                      |
| $(t_s)_S, (t_s)_E$         | Thickness of bond layer for surface bonded and embedded PZT patch, respectively                       |
| $T_3$                      | Stress in direction '3' (thickness of patch)                                                          |
| $T_p$                      | Axial stress in PZT patch                                                                             |
| $u(x,t)$                   | Displacement amplitude of dynamic vibration                                                           |
| $u_p$                      | Displacement at the interface between PZT patch and bond layer                                        |
| $u_b$                      | Corresponding displacement at interface between bond layer and beam                                   |
| $(u_{st})_o$               | Static deflection of beam at centre                                                                   |
| $(u_{st})_{peak}$          | Peak deflection of beam at centre                                                                     |
| $U$                        | Total energy generated by PZT patch                                                                   |
| $V_1$                      | Voltage across PZT patch in power measuring circuit                                                   |
| $V_2$                      | Voltage across Resistance ' $R_2$ ' in power measuring circuit                                        |
| $V_a$                      | Voltage output across accelerometer                                                                   |
| $V_F$                      | Voltage across piezo force-sensor                                                                     |
| $V_{FFT}$                  | FFT amplitude of voltage                                                                              |
| $V_L$                      | Voltage across load resistance attached to PZT patch                                                  |
| $(V_p)_S, (V_p)_E$         | Voltage across the terminals of surface bonded and embedded PZT patch, respectively                   |
| $(V_p^M)_S, (V_p^M)_E$     | Voltage generated by surface bonded and embedded PZT patch, respectively, considering mechanical loss |
| $(V_p^S)_S, (V_p^S)_E$     | Voltage generated by surface bonded and embedded PZT patch, respectively, considering shear lag loss  |
| $w_b$                      | Width of beam                                                                                         |
| $w_p$                      | Width of PZT patch                                                                                    |
| $x$                        | Displacement                                                                                          |
| $x'$                       | Real component of mechanical impedance                                                                |
| $\ddot{x}$                 | Acceleration                                                                                          |
| $X$                        | Reactance measured by AD5933                                                                          |
| $(X_{eff})_S, (X_{eff})_E$ | Effective length fraction of the surface bonded and embedded PZT patch, respectively                  |
| $y'$                       | Imaginary component of mechanical impedance                                                           |
| $y$                        | Beam deflection                                                                                       |

|                              |                                                                                   |
|------------------------------|-----------------------------------------------------------------------------------|
| $Y_o$                        | Amplitude of vibration                                                            |
| $\overline{Y^E}$             | Complex young's modulus of elasticity of the PZT patch at constant electric field |
| $(Y_{eff})_S, (Y_{eff})_E$   | Effective width fraction of surface bonded and embedded PZT patch, respectively   |
| $Z$                          | Mechanical impedance of the structural system,                                    |
| $Z_a$                        | Mechanical impedance of the PZT patch                                             |
| $\zeta$                      | Damping Ratio                                                                     |
| $\zeta_e$ and $\zeta_m$      | Transducer and mechanical damping ratio                                           |
| $\omega$                     | Operating angular frequency                                                       |
| $\omega_n$                   | Natural angular frequency                                                         |
| $\phi_n$                     | $n^{\text{th}}$ mode shape                                                        |
| $\theta$                     | Phase angle                                                                       |
| $\overline{\epsilon_{33}^T}$ | Complex electric permittivity (in direction '3') at constant stress               |
| $\zeta_E$                    | Strain lag ratio for embedded PZT patch                                           |
| $(\xi_E)_{L_p/2}$            | Strain lag ratio at the ends of embedded PZT patch                                |
| $\sigma_b$                   | Bending stress of beam at its extreme top fibre                                   |
| $\kappa$                     | Wave number.                                                                      |
| $\eta$                       | Mechanical loss factor of the PZT material                                        |
| $\eta_{cE}$                  | Compliance matrix loss factor                                                     |
| $\eta_S$                     | Structural loss factor                                                            |
| $\delta$                     | Dielectric loss factor of PZT material                                            |
| $\nu$                        | Poisson's ratio                                                                   |
| $\rho$                       | Density                                                                           |
| $\tau$                       | Interfacial shear stress                                                          |
| $\Gamma_S$                   | Shear lag parameter for surface bonded PZT patch                                  |
| $\Gamma_E$                   | Modified shear lag parameter for embedded PZT patch                               |
| $\gamma$                     | Shear strain in bond layer                                                        |

## **LIST OF ACRONYMS**

|       |                                                 |
|-------|-------------------------------------------------|
| AC    | Alternative Current                             |
| CVS   | Concrete Vibration Sensor                       |
| CVEH  | Concrete Vibration Energy Harvester             |
| c/c   | Centre to centre distance                       |
| DAQ   | Data Acquisition system                         |
| DC    | Direct Current                                  |
| DI    | Damage Index                                    |
| EMI   | Electro-Mechanical Impedance                    |
| FFT   | Fast Fourier Transformation                     |
| FRF   | Frequency Response Function                     |
| LCR   | Inductance Capacitance Resistance               |
| NASA  | National Aeronautics and Space Administration   |
| NDT&E | Non-destructive Testing and Evaluation          |
| PA    | Power Amplifier                                 |
| PVDF  | Polyvinylidene Fluoride                         |
| PZT   | Lead Zirconate Titanate (Piezoelectric Ceramic) |
| RC    | Reinforced Concrete                             |
| RF    | Radio Frequency                                 |
| RMS   | Root Mean Square                                |
| RMSD  | Root Mean Square Deviation                      |
| SBPS  | Surface Bonded PZT Sensor                       |
| SHM   | Structural Health Monitoring                    |
| SMIS  | Shuttle Modal Inspection System                 |
| SOI   | Silicon on Insulator                            |
| SSDL  | Smart Structures and Dynamic Laboratory         |
| UDL   | Uniformly Distributed Load                      |