VIBRATION CONTROL OF WIND TURBINE TOWERS: ANALYTICAL STUDEIS AND REDUCED SCALED EXPERIMENTS

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Vibration control of wind turbine towers: Analytical studies and reduced scaled experiments

by

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DEDICATION

I would like to dedicate this work to my late grandfather for making me the person I am today.

CERTIFICATE

This is to certify that the thesis titled **Vibration control of wind turbine towers: Analytical studies and reduced scaled experiments**, submitted by **Somya Ranjan Patro (2019CEZ8252)**, to the Indian Institute of Technology, Delhi, for the award of the degree of **Doctor of Philosophy**, is a bonafide record of the research work done by him under our supervision. The contents of this thesis, in whole or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

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ABSTRACT

The offshore wind turbine (OWT) is increasingly favored as a primary renewable energy option in numerous Asian nations due to its access to higher average wind speeds, decreased turbulence, reduced wind shear, and the advantage of not consuming as much land-based space. However, OWTs face challenging environmental conditions, including demanding geological circumstances, significant dynamic forces, and hazardous working conditions. Consequently, there is a critical need for vibration absorbers in offshore wind turbines, particularly when facing extreme loading conditions caused by wind and waves. This study aims to examine the dynamic performance of monopile-supported offshore wind turbine (MSWT) structures and analyze the vibration reduction characteristics of the entire system by employing the concepts of local resonance (LR) and rigid body dynamics (RBD).

The current research introduces a comprehensive analytical model of a monopile-supported offshore wind turbine (MSWT), considering non-uniform geometry and fluid-soil-structure interaction. The combined effect of rotor nacelle assembly and the blades are modeled as a rigid body placed eccentrically at the top of the tower. The tower and monopile are modeled using Euler-Bernoulli beam theory, with the tower's non-uniform geometry accounted for using the conicity parameter. The soil is modeled as visco-elastic springs connected throughout the length of the monopile. The effect of fluid-structure interaction is considered using the hydrodynamic mass. The force-displacement relationship between the tower and the monopile is established using the Spectral element method (SEM). Simplifying the entire MSWT system into a single-degree-of-freedom (SDOF) system, a closed-form expression for natural frequencies is derived to improve computational efficiency, validated against existing literature. Further, the SDOF system is subjected to random wind loads, and a tuned mass damper (TMD) controls the vibration. The random nature of the wind effects is simulated using the Kaimal Spectrum. The optimum parameters of the TMD are obtained using the H_2 optimization technique, where the objective function was defined as the standard deviation to be minimized. A near identity spectrum (NIS) has also been introduced to derive a closed-form mathematical formulation for the objective function. To obtain

insight into the dynamic characteristics of the TMD, small-scale experiments are conducted using accelerometers attached to a cantilever beam's free end, with results analyzed in the frequency domain via transmittance parameters. Furthermore, a novel omnidirectional TMD as a local resonator attached periodically throughout the tower has been proposed. The vibration attenuation characteristics of the novel TMDs are observed when subjected to wide frequencies of loads with the help of analytical and experimental studies. Parametric studies are undertaken to rationalize resonator mass and total numbers. Optimal parameters for the novel TMD under random wind and wave loads are determined using H_2 optimization. Building upon the insights gained from the aforementioned studies, another novel dynamic vibration absorber (DVA) has been proposed using the concept of rigid body dynamics (RBD). The proposed DVA is attached periodically throughout the tower, and the dynamic characteristics are obtained through transmittance parameters. The vibration attenuation characteristics of the proposed DVA are evaluated using both analytical and experimental studies. Furthermore, extensive parametric analyses were carried out to investigate how the proposed dynamic vibration absorber (DVA) performs with the ratio of its mass to that of the wind turbine. Moreover, the efficacy of the inclination angle is evaluated regarding the double anti-resonance dips (DAD).

The conicity and flexibility of the foundations play a major role in affecting the natural frequency of the entire system. The conicity is directly proportional to the stiffness of the MSWT, thus increasing the natural frequency significantly. Conversely, introducing flexibility into the foundation increases the system's time period, reducing its natural frequency. Additionally, the location of the TMD significantly controls the attenuation characteristics as it facilitates the occurrences of a lower frequency stop band. Incorporating multiple omnidirectional TMDs introduces stop bands across a frequency spectrum in the system, aligning closely with the natural frequency of each TMD. The different modes of vibration of omnidirectional TMD provide multiple anti-resonance drops, facilitating a wide range of attenuation. When maintaining a constant total mass of TMDs, the quantity and spacing of these dampers significantly impact the overall system's dynamic performance. Fewer TMDs with higher mass tend to exhibit a higher tuning ratio and lower damping ratio, and vice versa.

अमूर्त

ऑफशोर विंड टरबाइन कई एशियाई देशों में एक प्रमुख नवीकरणीय ऊर्जा विकल्प के रूप में तेजी से लोकप्रिय हो रहा है क्योंकि यह उच्च औसत पवन गति, कम अशांति, और कम पवन शियर तक पहुंच प्रदान करता है, और जमीन पर अधिक स्थान का उपयोग नहीं करता है। हालांकि, ऑफशोर विंड टरबाइन को चुनौतीपूर्ण पर्यावरणीय परिस्थितियों का सामना करना पड़ता है, जिसमें कठिन भूगर्भीय स्थितियां, महत्वपूर्ण गतिशील बल, और खतरनाक कार्य की स्थिति शामिल हैं। इसलिए, अपतटीय पवन टरबाइनों में कंपन अवशोषक की महत्वपूर्ण आवश्यकता होती है, खासकर जब हवा और तरंगों से उत्पन्न होने वाले चरम लोडिंग परिस्थितियों का सामना करना पड़ता है। इस अध्ययन का उद्देश्य मोनोपाइल-समर्थित अपतटीय पवन टरबाइन संरचनाओं के गतिशील प्रदर्शन की जांच करना और स्थानीय प्रतिध्वनि और कठोर शरीर गतिकी के अवधारणाओं को अपनाकर संपूर्ण प्रणाली की कंपन को कम करने के गुणों का विश्लेषण करना है।

वर्तमान अनुसंधान एक व्यापक विश्लेषणात्मक मॉडल प्रस्तुत करता है जो मोनोपाइल-समर्थित अपतटीय पवन टरबाइन की असमान ज्यामिति और तरल-भूमि-संरचना पारस्परिक क्रिया को ध्यान में रखता है। रोटर नैकले असेंबली और ब्लेड्स के संयुक्त प्रभाव को एक कठोर शरीर के रूप में मॉडल किया गया है जो टावर के शीर्ष पर विकेंद्रित रूप से रखा गया है। टावर और मोनोपाइल को यूलेर-बर्नौली बीम सिद्धांत का उपयोग करके मॉडल किया गया है, जिसमें टावर की असमान ज्यामिति को शंक्वाकार पैरामीटर का उपयोग करके माना गया है। मिट्टी को मोनोपाइल की पूरी लंबाई में जुड़ी हुई विवेकी स्प्रिंग्स के रूप में मॉडल किया गया है। तरल-संरचना पारस्परिक क्रिया के प्रभाव को हाइड्रोडायनामिक द्रव्यमान का उपयोग करके माना गया है। टावर और मोनोपाइल के बीच बल-विस्थापन संबंध को स्पेक्ट्रल एलिमेंट विधि का उपयोग करके स्थापित किया गया है। पूरे मोनोपाइल-समर्थित अपतटीय पवन टरबाइन सिस्टम को एकल-डिग्री-ऑफ-फ्रीडम सिस्टम में सरल बनाते हुए, प्राकृतिक आवृत्तियों के लिए एक बंद-रूप अभिव्यक्ति को प्राप्त किया गया है, जिससे गणनात्मक दक्षता में सुधार हुआ है, जिसे मौजूदा साहित्य के साथ सत्यापित किया गया है। इसके अलावा, एकल-डिग्री-ऑफ-फ्रीडम सिस्टम को यादृच्छिक पवन भार के अधीन किया गया है, और एक ट्यूनड मास डैम्पर कंपन को नियंत्रित करता है। पवन प्रभावों की यादृच्छिक प्रकृति को कैमल स्पेक्ट्रम का उपयोग करके अनुकरण किया गया है। ट्यूनड मास डैम्पर के अनुकूलतम पैरामीटर एच दो अनुकूलन तकनीक का उपयोग करके प्राप्त किए गए हैं, जहां उद्देश्य कार्य को न्यूनतम किए जाने के लिए मानक विचलन के रूप में परिभाषित किया गया था। उद्देश्य कार्य के लिए एक बंद-रूप गणितीय सूत्रीकरण प्राप्त करने के लिए एक निकट पहचान स्पेक्ट्रम भी पेश किया गया है। ट्यूनड मास डैम्पर की गतिशील विशेषताओं की अंतर्दृष्टि प्राप्त करने के लिए, छोटे पैमाने के प्रयोग किए गए हैं जिसमें एक्सेलेरोमीटर को एक कैंटिलीवर बीम के मुक्त सिरे पर जोड़ा गया है, और परिणामों का विश्लेषण ट्रांसमिटेंस पैरामीटर के माध्यम से आवृत्ति डोमेन में किया गया है। इसके अलावा, टावर की पूरी लंबाई में आवधिक रूप से जुड़े हुए स्थानीय प्रतिध्वनि के रूप में एक नए सर्वदिशात्मक ट्यूनड मास डैम्पर का प्रस्ताव किया गया है। विश्लेषणात्मक और प्रयोगात्मक अध्ययन की सहायता से व्यापक भार की आवृत्तियों के अधीन होने पर नए ट्यूनड मास डैम्पर के कंपन क्षीणन गुणों को देखा गया है। द्रव्यमान और कुल संख्या को युक्तिसंगत बनाने के लिए पैरामीट्रिक अध्ययन किए गए हैं। यादृच्छिक पवन और तरंग भार के तहत नए ट्यूनड मास डैम्पर के लिए अनुकूलतम पैरामीटर एच दो नुकूलन का उपयोग

करके निर्धारित किए गए हैं। उपरोक्त अध्ययनों से प्राप्त अंतर्दृष्टियों के आधार पर, कठोर शरीर गतिकी के अवधारणा का उपयोग करके एक अन्य नए गतिशील कंपन अवशोषक का प्रस्ताव किया गया है। प्रस्तावित गतिशील कंपन अवशोषक टावर की पूरी लंबाई में आवधिक रूप से जुड़ा हुआ है, और गतिशील विशेषताओं को ट्रांसमिटेंस पैरामीटर के माध्यम से प्राप्त किया गया है। प्रस्तावित गतिशील कंपन अवशोषक के कंपन क्षीणन गुणों का मूल्यांकन विश्लेषणात्मक और प्रयोगात्मक अध्ययन का उपयोग करके किया गया है। इसके अलावा, पवन टरबाइन के द्रव्यमान के अनुपात के साथ प्रस्तावित गतिशील कंपन अवशोषक के प्रदर्शन की जांच के लिए व्यापक पैरामीट्रिक विश्लेषण किए गए हैं। इसके अलावा, दोहरे प्रतिध्वनि विराम के संबंध में झुकाव कोण की प्रभावकारिता का मूल्यांकन किया गया है।

शंक्वाकारता और नींव की लचीलेपन की प्राकृतिक आवृत्ति को प्रभावित करने में महत्वपूर्ण भूमिका होती है। शंक्वाकारता सीधे MSWT की कठोरता के अनुपात में होती है, जिससे प्राकृतिक आवृत्ति में उल्लेखनीय वृद्धि होती है। इसके विपरीत, नींव में लचीलापन जोड़ने से प्रणाली की समय अवधि बढ़ जाती है, जिससे इसकी प्राकृतिक आवृत्ति में कमी आती है। इसके अतिरिक्त, ट्यूनड मास डैम्पर का स्थान क्षीणन गुणों को महत्वपूर्ण रूप से नियंत्रित करता है क्योंकि यह निचली आवृत्ति स्टॉप बैंड की घटनाओं को सुविधाजनक बनाता है। टावर में सर्वदिशात्मक ट्यूनड मास डैम्पर को शामिल करना प्रणाली में आवृत्ति स्पेक्ट्रम में स्टॉप बैंड पेश करता है, जो प्रत्येक ट्यूनड मास डैम्पर की प्राकृतिक आवृत्ति के साथ निकटता से मेल खाता है। सर्वदिशात्मक ट्यूनड मास डैम्पर के विभिन्न कंपन मोड्स कई प्रतिध्वनि ड्रॉप्स प्रदान करते हैं, जो व्यापक क्षीणन की सुविधा देते हैं। जब ट्यूनड मास डैम्पर का कुल द्रव्यमान स्थिर रखा जाता है, तो इन डैम्पर्स की संख्या और स्थान का समग्र प्रणाली के गतिशील प्रदर्शन पर महत्वपूर्ण प्रभाव पड़ता है। अधिक द्रव्यमान के साथ कम ट्यूनड मास डैम्पर एक उच्च ट्यूनिंग अनुपात और कम डैम्पिंग अनुपात प्रदर्शित करते हैं, और इसके विपरीत।

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ABBREVIATIONS

ACLD	Active Constrained Layer Damping
AMD	Active Mass Damper
ANN	Artificial Neural Network
ATMD	Active Tuned Mass Damper
BS	Bragg Scattering
BW	Band Width
CO_2	Carbon Dioxide
CPC	Collective Pitch Control
CY	Cycle Year
DAD	Double Anti-resonance Dip
DAQ	Data Acquisition System
DC	Direct Current
DFT	Discrete Fourier Transform
DSM	Dynamic Stiffness Matrix
DVA	Dynamic Vibration Absorber
EMSTMD	Electro-Magnetic Shunt Tuned Mass Damper
FBD	Free Body Diagram
FEM	Finite Element Method
FFT	Fast Fourier Transform
FSSI	Fluid Soil Structure Interaction
FSI	Fluid-Structure Interaction
GA	Genetic Algorithm
GDE	Governing Differential Equation
GHG	Greenhouse Gas
GW	Giga Watt
GWN	Gaussian White Noise
IFFT	Inverse Fast Fourier Transform

IPC	Individual Pitch Control
KS	Kaimal Spectrum
LQG	Linear Quadratic Gaussian
LQR	Linear Quadratic Regulator
LR	Local Resonance
MDOF	Multi-Degree of Freedom
MMC	Modified Mohr-Coulomb
MR	Magnetorheological
MSS	Mass Spring Sub-systems
MSWT	Monopile Supported Wind Turbine
MW	Mega Watt
NIF	Near Identity Spectrum
OTMD	Omnidirectional Tuned Mass Damper
OWT	Offshore Wind Turbine
PCBMR	Perpendicular Cantilever Beam-Mass Resonators
PLA	Poly Lactic Acid
PSD	Power Spectral Density
PZSD	Piezoelectric Shunt Damping
PTMD	Pendulum Tuned Mass Damper
RBD	Rigid Body Dynamics
RR	Response Reduction
RNA	Rotor Nacelle Assembly
SDG	Sustainable development goals
SDOF	Single Degree of Freedom
SEM	Spectral Element Method
SMR	Spring Mass Resonator
SMSDOF	Spring Mass Single degree of Freedom
SSI	Soil Structure Interaction
TID	Tuned Inerter Damper
TLD	Tuned Liquid Damper
TLCD	Tuned Liquid Column Damper
TRBD	Tuned Rolling Ball Damper

TRD	Tuned Rotor Damper
TTLCD	Toroidal Tuned Liquid Column Damper
TMD	Tuned Mass Damper
WTT	Wind Turbine Tower

NOTATIONS

$(\bullet)^I$	Partial differential concerning spatial coordinate
$\{\}^T$	Transpose of a vector/matrix
AC	Sensitivity of triaxial accelerometers in millivolt per meter per second square
AM	Sensitivity of uniaxial accelerometers in millivolt per meter per second square
A_j	j number of unknown arbitrary constants representing the transverse equation of motions of the tower
A_p	Cross-sectional area of the monopile in meter square
A_P	Cross-sectional area of the single elastic beam of the DVA in meter square
A_R	Cross-sectional area of the rotor in meter square
A_T	Cross-sectional area of the tower in meter square
b	Width of PLA beam in meters
b_e	Length of the single elastic beam of the DVA in meters
b_m	Length of the single rigid body of the DVA in meters
B	Lever arm of rotor overhang in meters
B_k	k number of unknown arbitrary constants representing the axial equation of motions of the tower
c	Constant for capturing variation in the cross-section, also known as conicity
c_1	Damping coefficient of the SDOF system
c_2	Damping coefficient of the TMD
c_d	Dashpot coefficients of the resonator
c_{dopt}	Optimum damping of the resonator
c_s	Dashpot coefficients to denote the visco-elastic nature of the spring-dashpot system
$\bar{c}$	Non-dimensional form of conicity
C_D	Drag coefficient
C_m	m number of unknown arbitrary constants representing the transverse equation of motions of the monopile submerged in water

C_T	Thrust coefficient
d_e	Thickness of the single elastic beam of the DVA in meters
d_m	Thickness of the single rigid body of the DVA in meters
D	Diameter of the rotor in meters
$\mathbf{D}_G$	Global displacement vector combining tower and monopile
D_o	o number of unknown arbitrary constants representing the transverse equation of motions of the monopile embedded in soil
D_p	Outside diameter of the monopile in meters
D_P	Outside diameter of the PLA tower in meters
D_{tb}	Bottom outside diameter of the tower in meters
D_{tt}	Top outside diameter of the tower in meters
e_x	Horizontal distance of the center of gravity of the rigid body from the tip of the tower in meters
e_y	Vertical distance of the center of gravity of the rigid body from the tip of the tower in meters
E_P	Young's modulus of the PLA in Newton per meter square
$E_P I_P$	Flexural rigidity of the PLA beam used in the experiment in Newton meter square
$E_P I_{PT}$	Flexural rigidity of the PLA tower used in the experiment in Newton meter square
$E_t I_t\ (x)$	Flexural rigidity of the tower in Newton meter square
$E_p I_p$	Flexural rigidity of the monopile in Newton meter square
f	Frequency of rotation of blades in Hertz
F	Fetch in meters
F_{cf}	Centrifugal force in Newton
F_D	Distributed wind force on the tower
$\mathbf{F}_G$	Global force vector combining tower and monopile
F_P	Nodal forces for single monopile element
$F_w\ (t)$	Wave force in time domain
F_T	Nodal forces for single tower element
F_{Th}	Thrust force in Newton
g	Acceleration due to gravity in meter per second square

h	Thickness of PLA beam in meter
H	Height of PLA tower in meters
H_f	Depth of the monopile submerged in water in meters
H_p	Total height of the monopile in meters
H_t	Length of the tower in meters
i	Complex integer iyota
I	Turbulence intensity
I_c	Inertia coefficient
I_m	Mass imbalance in kilogram meters
I_{min}	Performance index
$I_n(\varepsilon)$	nth order modified Bessel function of first kind
$\mathbf{I}_k$	Inertial force matrix of the k^{th} unit cell of the DVA
J	Polar moment of inertia of the rigid body in meters to the fourth power
$J_n(\varepsilon)$	nth order Bessel function of first kind
k	Stiffness coefficient of resonator in Newton per meters
k_e	Equivalent spring stiffness of the classical resonator in Newton per meter
k_{opt}	Optimum spring stiffness of the TMDs
k_s	Stiffness coefficient of the spring-dashpot system used to model soil in Newton per meter
K_e	Equivalent stiffness in Newton per meter
$K_n(\varepsilon)$	nth order modified Bessel function of second kind
K_L	Lateral spring stiffness in Newton per meter
K_R	Rotational spring stiffness in Newton meter per radian
$[\mathbf{K}]_{6\times6}$	Combined dynamic stiffness matrix considering the tower as well as the RNA
l_e	Length of the single elastic beam of the DVA in meters
l_m	Length of the single rigid body of the DVA in meters
L	Length of PLA beam in meters
L_i	Length of each element of PLA tower in meters
L_k	Integral length scale
L_P	Length of each element of monopile in meters
L_t	Total Length of PLA tower in meters
L_T	Length of each element of the tower in meters

L_W	Length of PLA blade in meters
m	Mass of the resonator in kilogram
m_1	Mass of the SDOF system in kilogram
m_2	Mass of the TMD in kilogram
m_a	Mass of the TMD used in the experiment in kilogram
m_d	Ratio of mass of the rigid body of the DVA to the mass of accelerometer
m_j	Mass of the j^{th} node of the DVA in kilogram
m_R	Equivalent lumped mass of blades in kilogram
m_s	Mass of the spring used in the experiment in kilogram
$m\left(x_f\right)$	Hydrodynamic mass, which represents the lateral resistance of the water-pile interface
M	Bending moment component of the PLA beam used for the experiment in Newton meter
M_a	Mass of the uniaxial accelerometer in kilogram
M_{acc}	Mass of the triaxial accelerometer in kilogram
M_c	Mass of the classical resonator in kilogram
M_e	Equivalent mass in kilogram
M_D	Mass of the rigid body of each DVA in kilogram
M_f	Bending moment component of the monopile submerged in water in Newton meter
M_{ft}	Mass of the force transducer in kilogram
M_i	Bending moment component of each component of the PLA tower in Newton meter
M_L	Mass of the rotor nacelle assembly used in the experiment in kilogram
M_m	Ratio of mass of all resonators to the combined mass of tower and the RNA
M_P	Mass of the concrete block in kilogram
M_{pi}	Bending moment component of each component of the monopile in Newton meter
M_R	Mass of the rotor nacelle assembly in kilogram
M_s	Bending moment component of the monopile embedded in soil in Newton meter
M_t	Bending moment component of the tower in Newton meter

M_w	Nodal bending moments acting on submerged monopile
M_x	Side-side Bending moment acting along the x-axis in Newton meter
M_y	Fore-aft Bending moment acting along the y-axis in Newton meter
n	Total number of resonators
N_p	Total number of monopile elements
N_P	Shape functions for monopile element
N_t	Total number of tower elements
N_T	Shape functions for tower element
N_w	Total number of monopile elements submerged in water
P	Axial force component of the tower in Newton
P_f	Peak enhancement factor
R	Radial distance from the center of the rotor in meters
r_b	Bottom outside radius of the tower in meters
r_p	Outside radius of the monopile in meter
r_t	Top outside radius of the tower in meters
$\mathbf{S}^i$	Dynamic stiffness matrix of i^{th} elastic beam element of DVA
S_I	Intensity of the spectrum
$[\mathbf{S}]_{6\times6}$	Dynamic stiffness matrix of the tower having six rows and six columns
$\left[\mathbf{S}_f\right]_{4\times4}$	Dynamic stiffness matrix for the monopile element submerged in water
$S_{FF,waves}\left(\omega\right)$	Power spectrum of wave force exerted on monopile
$S_{FF,wind,k}(\omega)$	The spectral density of turbulent thrust force acting on the rotor
$[\mathbf{S}_R]_{6\times6}$	Dynamic stiffness matrix for a beam having tip mass and a resonator at an arbitrary location
$[\mathbf{S}_G]_{10\times10}$	Global dynamic stiffness matrix combining tower and monopile having 10 rows and 10 columns
$S_{uu,k}\left(\omega\right)$	Theoretical Kaimal Spectrum for fixed point reference points in space as a function of excitation frequency
$S_{ww}\left(\omega\right)$	Power spectral density function of ocean wave
$\tilde{S}_{uu,k}(\omega)$	Normalized Kaimal spectrum
$S_{\omega_x\omega_y}\left(\omega, x\right)$	Spectral density of turbulent gust drag force
t	Time in seconds
$\bar{t}$	Non-dimensional form of time

t_p	Thickness of the monopile section in meters
t_B	Thickness of the PLA blade in meters
t_P	Thickness of the PLA tower in meters
T_0	Allowable transmittance
T_K	Kinetic energy in Joules
T_r	Analytical transmittance
T_R	Experimental transmittance
$u(x,t)$	Axial displacement of the tower
$\bar{U}$	Mean wind speed in meters per second
$U(x)$	Wind velocity as a function of tower depth in meters per second
$\bar{U}_{ref}$	Average wind velocity at a reference height in meters per second
$v(z,t)$	Fluctuating wind velocity as a function of tower depth and time in meters per second
v_s	Fluctuating wind velocity at hub height in meters per second
V	Shear force component of the PLA beam used for the experiment in Newton
V_f	Shear force component of the monopile submerged in water in Newton
v_h	Mean wind velocity at hub height in meters per second
V_i	Shear force component of each element of the PLA tower in Newton
V_{pi}	Shear force component of each element of the monopile in Newton
V_{rotor}	Velocity of the rotor in revolutions per minute
V_s	Shear force component of the monopile embedded in soil in Newton
V_t	Shear force component of the tower in Newton
V_w	Nodal shear forces acting on submerged monopile
$w(x,t)$	Transverse displacement of the PLA beam used for experiment
$w_b(t)$	Base excitation
$w_f(x_f,t)$	Transverse displacement of the monopile submerged in water
$w_i(x_i,t)$	Transverse displacement of each component of PLA tower
$w_{pi}(x_{pi},t)$	Transverse displacement of each component of monopile
$w_s(x_s,t)$	Transverse displacement of the monopile embedded in the soil
$w_t(x,t)$	Transverse displacement of the tower
$w_T(x,t)$	Transverse displacement of the PLA tower
$\bar{w}(\bar{x},\bar{t})$	Non-dimensional form of transverse displacement of the tower
W_B	Width of the PLA blade in meters

x	Co-ordinate axis of the tower in meters
x_D	Co-ordinate axis of i^{th} elastic beam of the DVA in meters
x_f	Co-ordinate axis of monopile submerged in water in meters
x_s	Co-ordinate axis of monopile embedded in soil in meters
$\bar{x}$	Non-dimensional form of space
x_{ref}	Reference height in meters
x_i **and** y_i	Distance between master node and slave node in local co-ordinates of the rigid body of the DVA
X_i **and** Y_i	Distance between master node and slave node in global co-ordinates of the rigid body of the DVA
$y(t)$	Displacement of the resonator
$y_{normGWN}$	L_2 norm or root mean square of the displacement responses of the TMD system optimized by Gaussian white noise
y_{normKS}	L_2 norm or root mean square of the displacement responses of the TMD system optimized by Kaimal Spectrum
$Y_n(\varepsilon)$	nth order Bessel function of second kind
α	Wave numbers for the monopile element submerged in water
β	Wave numbers for the monopile element embedded in soil
δ	Simplified non-dimensional form of natural frequency
η	Frequency ratio
$\eta_c(\bar{x})$	Concity ratio
η_{rat}	Ratio of the natural frequency of flexible support system to that of fixed support system
γ	Ratio of the mass of the RNA to that of the mass of the tower
γ_k	Stiffness correction factor
γ_m	Mass correction factor
κ_L	Non-dimensional lateral spring stiffness coefficient
κ_R	Non-dimensional rotational spring stiffness coefficient
λ	Non-dimensional excitation frequency
λ_D	Wave number of transverse direction for i^{th} elastic beam of the DVA
Λ	Equivalent area in meter square
μ	Ratio of mass of TMD to that of mass of SDOF system

μ_D	Wave number of axial direction for i^{th} elastic beam of the DVA
$\mu_{\min}$	minimum level of attenuation achieved within the double anti-resonance dips within the frequency range $\Omega_{\min}$
μ_s	Poisson's ratio of soil
ν	Ratio of the natural frequency of the TMD to that of the SDOF system
ν_{opt}	Optimum tuning ratio
ω	Excitation frequency in Hertz
ω_1	Natural frequency of the SDOF system in Hertz
ω_2	Natural frequency of the TMD in Hertz
ω_w	Wave number
ω_p	Peak frequency in Hertz
ω_r	Natural frequency of the DVA in Hertz
$\bar{\omega}$	Natural frequency of omnidirectional resonator in Hertz
Ω	Rotational frequency in radians per second
$\Omega_{\max}$	Conventional maximum bandgap width
$\Omega_{\min}$	Bandgap width for minimum negative transmittance
$\bar{\Omega}$	Angular frequency in radians per second
ψ	Phase angle
ϕ_i	Angle between i^{th} elastic beam local axis and the global axis
$\rho_t A_t(x)$	Mass per unit length of the tower in kilogram per meter
$\rho_p A_p$	Mass per unit length of the monopile in kilogram per meter
$\rho_P A_P$	Mass per unit length of the PLA beam used for the experiment in kilogram per meter
ρ_w	Water density in kilogram per cubic meter
ρ_s	Soil density in kilogram per cubic meter
ρ_a	Air density in kilogram per cubic meter
ρ_P	Density of PLA in kilogram per cubic meter
σ_U	Standard deviation of mean wind speed
σ_{xx}^2	Standard deviation of displacement response
τ	Thickness of the tower section in meters
θ	Rotation component of the PLA beam used for experiment
θ_d	Inclination angle of i^{th} elastic beam with the horizontal in degrees

θ_f	Rotation component of the monopile submerged in water
θ_i	Rotation component of each element of the PLA tower
θ_{pi}	Rotation component of each element of the monopile
θ_s	Rotation component of the monopile embedded in soil
θ_t	Rotation component of the tower
φ	Inclination angle of blades from vertical in degrees
ϖ **and** ω_n	Arbitrary constants bearing the units of dimensional quantity
ς	Wind shear (or power law) exponent
Ξ	Amplitude
ζ_1	Damping ratio of the SDOF system
ζ_2	Damping ratio of the TMD
ζ_{2opt}	Optimum damping ratio