SOME APPROXIMATIONS FOR FILTER DESIGN

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CERTIFICATE

This is to certify that the thesis entitled, 'SOME APPROXIMATIONS FOR FILTER DESIGN' being submitted by P. Varanasi, to the Department of Electrical Engineering, Indian Institute of Technology, Delhi, for the award of the degree of Doctor of Philosophy, is a record of banafide research work carried out by him under my supervision and guidance and in my opinion, it has reached the standard fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted to any other institute for the award of any degree or diploma.

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ABSTRACT

This thesis presents some new rational function approximations for filters in which magnitude and transient response or magnitude and group delay response are simultaneously specified. The new approximations possess superior characteristics and/or specific advantages over the existing ones, and may be categorized as (i) transitional Butterworth-Chebyshev approximations; (ii) minimum phase approximations with flat delay in the passband and equiripple magnitude in the pass- and stop- bands; (iii) bandpass approximations with arithmetically symmetrical magnitude and delay characteristics. Also presented in the thesis is a simple method for obtaining the parameters of a few important approximations of the transitional Butterworth-inverse Chebyshev type.

The transitional Butterworth-Chebyshev approximations serve to achieve, simultaneously, a steeper cutoff than in a Butterworth and a smaller transient response overshoot than in a Chebyshev. Based on the observation that the essential difference between these two approximations lies in the number of points of flatness of the magnitude characteristic, a new class of transitional Butterworth-Chebyshev approximations is derived. These are almost as simple as the Chebyshev approximations and their poles, as shown in this thesis, can be expressed in closed form; consequently neither their formulation nor pole evaluation requires the aid of a computer

as is the case with the earlier varieties. The new approximations are characterized by the maximum possible cutoff slope and an equiripple passband magnitude.

Maximally flat delay all pole filters which are derived from a truncated continued fraction of coth s can be made to have a single ripple in the passband magnitude by a suitable choice of the last coefficient of the approximant of coth s. This increases selectivity and reduces the degree of flatness by 2. Further, given the denominator polynomial of a transfer function, the numerator which results in an equiripple stopband magnitude can be obtained numerically using a certain frequency transformation. Combining these two known techniques and using the Newton-Raphson method, minimum phase transfer functions which possess (1) a flat delay characteristic, and (2) a magnitude characteristic with a single ripple in the passband, and the maximum possible number of equal ripples in the stopband, are derived. These functions possess enhanced selectivity and require, for given magnitude and delay specifications, a lower order than the known minimum phase approximations.

Based on the mirror image symmetry observed between the characteristics of the Matthaei and conventional bandpass filters, it is shown that the two filters can be combined to obtain bandpass filters with very good arithmetic symmetry of magnitude as well as delay. This technique is simple and straightforward and, unlike the earlier ones, involves neither numerical approximations nor spurious passbands.

The thesis also presents a simple semi-analytical method for obtaining the parameters of the transitional Butterworth-inverse Chebysheve type of filters with only one or two pairs of transmission zeros; such filters possess delay and magnitude characteristics comparable to those of the inverse Chebyshev without being as complex and are therefore attractive alternatives to the latter.

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