SOME GENERALIZED DISCRETE AND q-DISCRETE PROBABILITY DISTRIBUTIONS

By

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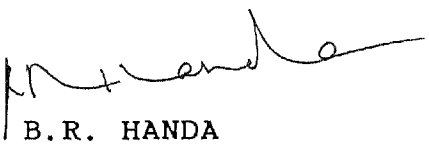
Vaishnomataarpanam

CERTIFICATE

This is to certify that the thesis entitled "SOME GENERALIZED DISCRETE AND q-DISCRETE PROBABILITY DISTRIBUTIONS" which is being submitted by SUNGEETA SINGH for the award of DOCTOR OF PHILOSOPHY (MATHEMATICS) to the Indian Institute of Technology, Delhi, is a record of bonafide research work carried out by her under my guidance and supervision. The thesis has reached the standard fulfilling the requirements of the regulations relating to the degree. The results obtained in the thesis have not been submitted to any other University or Institute for the award of any degree or diploma.



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ABSTRACT

OF

THE THESIS

ENTITLED

SOME GENERALIZED DISCRETE AND q- DISCRETE PROBABILITY DISTRIBUTIONS

Discrete models are used to represent processes where the output of interest is a count or enumeration. Continuous efforts have been made by researchers, through statistical and probability modelling, to study various discrete models and its properties which are of practical interest in various applied fields. The basic building block of many standard discrete distributions is the Bernoulli trial. In this thesis, discrete distributions based on Bernoulli trials with variable probabilities of success also known as Poisson trials are studied. Two special forms of these variable probabilities of success, one known as Log Linear Odds (LLO) probability and the other Log Linear (LL) probability, have been considered in order to make the models based on Poisson trials mathematically tractable. We call the trials with LLO or LL probability as q-Bernoulli trials so that when $q \to 1$, both of these reduce to Bernoulli trials. The corresponding distributions are called q-discrete distributions, which reduce to their ordinary counterparts as $q \to 1$. Besides studying several new properties of the known q-Binomial and q-Logarithmic series distributions, an attempt has been made in this thesis to define and study the q-Negative Binomial distribution, q-Hypergeometric distribution and

q-Negative Hypergeometric distribution in some details. Also a numerical method of moments estimation procedure has been developed for known and new q-distributions and illustrative data sets have been fitted. Also some new classes of q-discrete distributions have been defined and their properties which include moments, generating functions, some characterizations and limiting properties have been established.

Apart from the q-discrete distributions, two different generalizations of probability distributions based on first passage events and associated with Bernoulli trials with constant probability of success have been studied. The first one uses independent Bernoulli trials and gives rise to discrete distributions commonly studied under the title of distributions of order k. A Generalized Negative Binomial Distribution of order k, its Multiparameter and Multivariate Generalizations have been studied alongwith their moment and limiting properties.

The second generalization uses sequences of independent Bernoulli trials with the r.v. at the i^{th} trial taking value $-\mu_i$ or 1 with probability q or p respectively. The first passage probability to a given position on the random walk defined on this sequence of Bernoulli trials give rise to a class of multiparameter discrete distributions. The moment properties of particular cases of this class are discussed.

CONTENTS
