A THESIS ON

SOME THEORETICAL PROBLEMS OF GASDYNAMICS AND MAGNETOGASDYNAMICS

By

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C E R T I F I C A T E

This is to certify that the thesis entitled "Some theoretical problems of gasdynamics and magnetogasdynamics", being submitted by Mr. S.K. Majumdar to the Indian Institute of Technology, New Delhi, for the award of the Degree of Doctor of Philosophy in Mathematics, is a record of bonafide research work carried out by him. Mr. S.K. Majumdar has worked under my guidance and supervision and has fulfilled the requirements for the submission of this thesis, which to my knowledge, has reached the requisite standard.

The results contained in this thesis have not been submitted, in part or in full, to any other University or Institute for the award of any degree or diploma.

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PREFACE

The stellar and terrestrial atmospheres present a variety of interesting phenomena. One such is the high temperature of the solar corona with consequent coronal expansion which, it seems, may influence the temperature structure of the topmost layer of our atmosphere.

The phenomena of force-free magnetic fields in the envelope of some stars is another which has recently attracted the attention of astrophysicists.

The terrestrial atmosphere is now being thoroughly investigated by means of rockets and satellites. The phenomenon of turbulence in the ionospheric region successfully explains some interesting observations on the expansion of artificial clouds.

This thesis is a study of some particular aspects of the problems outlined above.

CONTENTS

PART - I

Thermal heating of the solar chromosphere and corona

S Y N O P S I S

This thesis deals with some theoretical problems of gas-dynamics and magnetogasdynamics. It is divided into two parts. The first part concerns itself with a certain astrophysical problem; the problem of thermal heating of the solar chromosphere through shock waves and the explanation of the temperature profile in the chromosphere. We have formulated the problem as an initial value problem of the one-dimensional gas flow equations taking heat conduction and radiation into account and have suggested a numerical method (using a new independent variable) for the solution of the above problem. We present a few numerical results illustrating the use of the new variable through graphs and tables.

In the second part, analytical solutions have been obtained for two related astrophysical problems. Two problems on magneto-gasdynamics and a problem on turbulent diffusion are also discussed in this part.

The first chapter of Part I starts with an introduction to the problem of the chromosphere heating, reviewing some of the important works in this field. The problem has two aspects. First, we must identify the source of energy input in the chromosphere. Secondly, one should enquire how this energy distributes itself to give rise to the observed temperature profile in the chromosphere corona. As regards the source of energy input, the hypothesis of Biermann and Schwarzschild that acoustic waves originating in the hydrogen-convection zone of the sun convert themselves into shock in the chromosphere and heat up the chromosphere and corona forms the basis of this work. Hence after this introduction we discuss the propagation of acoustic waves in a gas with (i) uniform temperature (ii) non-uniform temperature. For uniform temperature, we seek a solution in which the amplitude of the disturbance increases exponentially with height, the gas being at rest under gravity. For non-uniform temperature distribution, we have obtained a general expression for the velocity of the disturbance in a closed form using characteristic co-ordinates. In this case the equation governing the velocity of the disturbance has been shown to be of Darboux's form. It is also found that temperature should decrease uniformly with height if the gas is at rest under gravity.

The second chapter formulates the physical problem of chapter I mathematically. We have in view the second aspect of the problem namely, how the input energy (delivered through shock) distributes itself and gives rise to the observed temperature profile. With this aim we have written down the basic equations of one-dimensional gas flow using as one of the independent variables a certain "mean density". We show that the equation governing the acoustic disturbance of the previous chapter remain invariant under the use of the mean density as a new independent variable and derive the equation giving x as a function of mean density and time. This is the fourth equation to be solved numerically along with the equations of momentum, continuity and energy. The four characteristics of these equations are found by writing them in a matrix form suggested by Lax. We also analyse the terms representing heat conduction and radiation and have suggested a new expression to represent radiative energy loss. Attention is also focussed in deriving the initial conditions of the problem in an altogether new fashion. We suggest the use of a discontinuous function to represent initial temperature distribution. This is followed by the derivation of boundary conditions. This chapter closes with a few critical remarks on the criteria of well-posed problems as applied to the present situation.

The third chapter gives the derivation of the difference equations corresponding to the basic equations, the boundary and the initial conditions of the previous chapter using the artificial viscosity term followed by a discussion on the stability of the difference equations and the parameters of the problem. The difference equations incorporating artificial viscosity term are set up according to the scheme of Lelevier and are solved by iteration with mesh length.1. In view of the tremendous amount of work involved, it is necessary to use the electronic digital computer IBM 7044 (installed at I.I.T., Kanpur). To illustrate the use of the mean density as an independent variable the numerical integration is carried out for the case of the propagation of acoustic wave in an isothermal atmosphere (without radiation and heat conduction). The results show that density and temperature increase whenever a condensation pulse propagates through the medium and fall with the advent of a rare-faction pulse. This is quite consistent with our expectation and justifies the use of mean density as an independent variable. We have presented graphically as well as by tables the propagation of the acoustic wave in a medium at rest under gravity for five different values of the parameter ($\bar{\omega}$) characterising the mean density.

The second part consists of three chapters. The first chapter discusses the following two problems :(1) the temperature distribution in the chromosphere corona transition region assuming that heat conduction and radiation are the only two processes at work and the source of heat is the conductive flux from the base of the corona, (2) the extension of the result of temperature distribution in the corona (Chapman) by taking radiative heat loss into account. For radiative heat loss we use the result obtained in the first part of this work. We equate the terms representing heat conduction and the radiation in a static spherically symmetrical chromosphere and corona. In the first case the temperature in the transition region increases with r attaining the maximum value at the coronal base. In the second case as we start from the coronal base and go outwards, the fall in temperature is less rapid than that in the absence of radiative cooling.

The first part of the second chapter investigates a problem of force-free magnetic fields. A magnetohydrostatic field is found which becomes force-free due to suitable displacement of the medium. The geometry of the problem is a rectangular box and the solutions satisfying the boundary conditions are found to be trigonometric functions. An expression for the various modes of displacement is also found.

In the second part, dimensional considerations are applied to justify the neglect of the induced radial magnetic field by Stewartson in his treatment of the problem of the rotating disk in the presence of a magnetic field. It is found that if the depth of the liquid column is small compared to the radius of the rotating disk, radial magnetic field has negligible effect on the motion.

The last chapter of Part II gives both qualitative and quantitative discussion of the role of turbulent diffusion in the earth's upper atmosphere in explaining the experimental observations on the diffusion of chemical clouds by rockets. An attempt is made to explain the growth of chemical cloud releases at altitudes between 100 - 160 km., using the theory of turbulent diffusion. After recapitulating some of the relevant concepts of turbulent diffusion a formulae based on these concepts are applied to fit the experimental data. A fairly good agreement between theoretical and experimental curves suggests eddy diffusion as the major transport process at certain altitudes. The variation of eddy scales with height is briefly discussed and the observed diffusion coefficients of sodium and cesium are examined in the light of this theory.