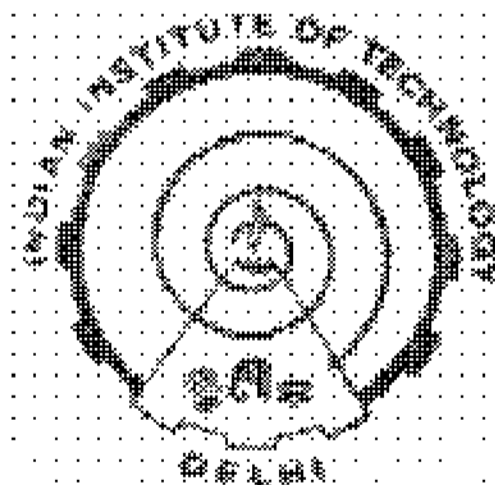


**A CRITICAL ASSESSMENT OF CTOD IN SMALL SCALE
YIELDING AND ELASTIC-PLASTIC CRACK GROWTH SITUATIONS**

by
C. R. PRATAP

Submitted
in fulfilment of the requirements
for the degree of
DOCTOR OF PHILOSOPHY



Department of Applied Mechanics
INDIAN INSTITUTE OF TECHNOLOGY, DELHI
1984

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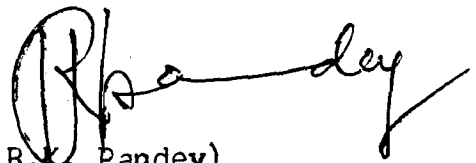
TO

MY MOTHER

WITH LOVE AND DEDICATION

CERTIFICATE

This is to certify that the Thesis entitled "A Critical Assessment of CTOD in Small Scale Yielding and Elastic-Plastic Crack Growth Situations" being submitted by C.R. Pratap to the Indian Institute of Technology, Delhi, India, for the award of the degree of Doctor of Philosophy in Applied Mechanics Department is a record of bonafied research work carried out by him under my supervision and guidance. The Thesis work, in my opinion, has reached the standard fulfilling the requirements for the Doctor of Philosophy Degree. The research report and results presented in this Thesis have not been submitted in part or in full to any other University or Institute for the award of any degree or diploma.



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C.R. Pratap

ABSTRACT

Crack tip opening displacement (CTOD) has been studied as an elastic-plastic fracture parameter. CTOD has been related to stress intensity factor (K) in Linear Elastic Fracture Mechanics (LEFM) through the factor m' . There appears to be a controversy as regards to the value of m' in K-CTOD relationship. In Elastic-Plastic Fracture Mechanics (EPFM), CTOD has been obtained from clip gauge displacement by making use of hypothetical plastic hinge model and rotational factor, r . However, the models for CTOD determination using rotational factor, r are not capable of handling all dimensional parameters, and loading geometry of the specimen. Also a controversy exists in selecting the r value because the r is now believed to depend on the geometry and material property. Furthermore, the role of state of stress vis-a-vis specimen dimensions on CTOD, J-integral, the K-CTOD and J-CTOD relationship has not yet been fully established.

The present investigation has been conducted keeping in mind the above problems associated with the CTOD evaluation.

Chapter-I deals with the background of CTOD, and its usage as an elastic-plastic fracture parameter. Some of the difficulties encountered in the evaluation of CTOD and in its characterization as a fracture parameter are outlined.

Chapter-II is a review of available literatures concerning the definition and determination of CTOD in LEFM and EPFM situations and the problems concerning variation of m' and r with specimen dimensions loading geometry and material property. A review of J determination in slow stable crack growth has also been presented. The variation of M in J-CTOD relation with specimen dimension and material property has been identified. The chapter finally concluded with the importance of searching out a suitable model of CTOD and to prove its validity in small scale yielding as well as in elastic-plastic crack growth situations.

Chapter-III presents an analytical model for CTOD determination in small scale yielding and elastic-plastic crack growth situations, using the plastic zone size formed at the crack tip. A composite nature of crack profile characteristic of an edge cracked specimen (as indicated by FEM solutions) has been adopted in the analysis. The plastic zone size at the crack tip has been estimated from the experimental load-crack mouth opening displacement diagram on the line of ASTM recommendations of effective crack length determination. An equation to evaluate r has also been proposed.

Chapter-IV provides details of test specimens, necessary instrumentation, and the experimental set-up used in the present investigation. The test procedures for K , CTOD and J determination, crack growth evaluation, plastic zone measurement are also given.

Chapter-V gives an account of the results obtained in the present investigation in small scale yielding situation as defined in this work. The results have been obtained by processing the experimental data through the computer and by using the proposed model.

Chapter-VI presents the discussion of the results obtained in Chapter-V. The validity of the proposed model for the determination of CTOD has been assessed by using the existing models (like Wells', Dawes', Hollstein et al.'s models). The present model, appears to be workable in all the specimen dimensions ($B, a/W$) and loading geometry and not restricted to only deep crack bend or CT specimens. The state of stress has been represented by effective plastic constraint, m_e and this also has been found to be geometry dependent. The rotational factor, r has been evaluated by the proposed method in this work and compared with the available methods in the literature. The variation of r with specimen and loading geometry has also been presented. The m' and M have been shown to be geometry dependent. The effect of thickness, a/W and loading geometry on CTOD at crack initiation has been discussed.

Chapter-VII deals with the experimental results in elastic-plastic crack growth situation. The method of obtaining the crack growth, Δa from a single specimen test is presented. The derived CTOD-resistance curve and J_R -curve have been obtained by the model presented in this investigation

as well as by existing models. The results have been presented upto the maximum load point.

Chapter-VIII is concerned with the discussion of results of Chapter-VII. The validity of the proposed CTOD model for determination of CTOD-resistance curve in slow stable crack growth situation has been examined. The proposed model is justified by comparing it with the Wells' method for deep bend and Hollstein et al.'s method for CT specimens. The $CTOD_R$ values obtained at the maximum load point are found to be geometry dependent. J_R curve has also been evaluated by using the rotational factor obtained from the proposed model and the resulting J_R values have been compared with Garwood et al.'s and Rice's methods. The M values at crack initiation and maximum load points are found to be geometry dependent.

Chapter-IX shows the application of the present concept of CTOD determination in notched bars. A mathematical analysis for determination of notch mouth opening displacement (NMOD) and notch tip opening displacement (NTOD) is presented. For plane strain plastic zone calculation in tension bars, a modified version of Smith's model has been applied. NTOD from NMOD was obtained for SEN tension and bend specimens. A discussion on the variation of plastic zone size and NTOD in above geometries has been presented.

The conclusions based on the present investigation are presented in Chapter-X.

LIST OF TABLES

Table No.	Description	Page
4.1	General composition of IS 226 steel	67
4.2	Mechanical properties of IS 226 steel	67
5.1	Plastic zone size at crack initiation point	94
6.1	K_T and K_I values in different specimens	113
6.2	The CTOD values at the crack initiation point obtained by different methods	128
6.3	The values of m' , m_e and p in different specimens	135
6.4	The rotational factor at the crack initiation point (r_i) in different geometries	143
6.5	The values of J and M at crack initiation point in some representative specimens	143
8.1	Typical general yielding loads for specimens of different geometries	164
8.2	J-integral at crack initiation and maximum load point	164
8.3	Comparison of J-integral values evaluated by different methods	181
8.4	Variation of M in different specimens with crack extension	183
8.5	M at the maximum load point in different specimens	183

LIST OF FIGURES

Fig. No.	Description	Page
2.1	Crack tip stress field: components and coordinates	10
2.2	Arbitrary contour for J-integral	10
2.3	Schematic load (P)-load line displacement (q) diagram	10
2.4	Hypothetical model for defining CTOD	10
2.5	A schematic diagram of horn shaped plastic zone	10
2.6	The effective and the nominal plastic zone size in a precracked body	21
2.7	Plastic hinge model for CTOD determination	21
3.1	Infinite centre crack plate	43
3.2	A finite width SEN specimen	43
3.3	Proposed model for determination of CTOD by considering a composite crack profile	51
3.4	The position of the common point C in three cases: (i) $C_x < a$, (ii) $C_x = a$ and (iii) $C_x > a$	51
3.5	A schematic load-CMOD diagram showing various components of CMOD	61
3.6	Definition of r in SSY situation	61
3.7	Definition of r in slow crack growth situation	61
4.1	Identification of specimen orientation	69
4.2	Fracture mechanics test specimens	70
4.3	Uniaxial tensile test piece	70
4.4	Experimental set-up: A schematic diagram	73
4.5	Details of contraction gauge	75

LIST OF FIGURES (Contd.)

Fig. No.	Description	Page
4.6(a)	An experimental set-up for fracture testing	77
4.6(b)	A close up view of the set-up	78
5.1	Effect of (a) a/W ratio and (b) thickness on load CMOD diagrams for SEN bending specimens	85
5.1(c)	Load-CMOD diagrams for different loading geometries	87
5.2	Diagram showing relation between load line displacement and CMOD	87
5.3(a)	Effect of thickness and a/W ratio on P-q diagram in SEN bending specimen	89
5.3	Effect of (b) thickness in tension and (c) loading geometry on load-load line displacement diagrams	90
5.4	Schematic diagram showing the shape of plastic zone in different specimen geometries	93
5.5(a)	Comparison of CTOD values obtained by different methods	98
5.5	Comparison of CTOD by different methods in (b) shallow crack SEN bending and (c) deep crack CT specimens	99
5.6	Effect of (a) thickness and (b) a/W ratio on CTOD obtained in the present investigation (eqn. 3.36)	100
5.6(c)	Effect of specimen geometry on CTOD obtained in the present investigation (eqn. 3.36)	101
5.7	Effect of (a) Specimen Thickness and a/W ratio in SEN bending specimen and (b) loading geometry on notch root contraction	102
5.8	The variation of rotational factor with CTOD in SSY regime	104

LIST OF FIGURES (Contd.)

Fig. No.	Description	Page
5.8(c)	The effect of yield strength on rotational factor, r in SEN bending specimen	106
5.9	Deformed grid patterns at the crack tip	107
5.10	Distribution of displacements and strains at the crack	108
6.1	Variation of C_x with K in specimens with different geometries and loading configurations	112
6.2	Variation of m_e with thickness in (a) deep crack SEN bending and in (b) shallow crack SEN bending specimens	116
6.2(c)	Variation of m_e with thickness in different loading geometries	119
6.3	Variation of m_e with a/W ratio	119
6.4	Effect of loading geometry on effective plastic constraint	121
6.5	Effect of loading geometry on CTOD values	121
6.6	Comparison of rotational factor, r based on proposed model with plastic hinge model in SSY situation	131
6.7(a)	Variation of m' with K for different materials	131
6.7(b)	Variation of m' with K normalised to yield strength of the material	137
6.8	On-load NRC vs on-load CTOD	137
6.9	J as a function of load line displacement in SEN bending and SEN tension specimens	145
6.10	Comparison of K_J with K_{eff} values	147
7.1	Effect of (a) thickness and (b) a/W ratio on load-CMOD and load-crack growth diagrams	151
7.1(c)	Load-CMOD and crack growth diagrams for different loading geometries	152

LIST OF FIGURES (Contd.)

Fig. No.	Description	Page
7.2	A schematic representation of load-CMOD diagram with unloading curve	154
7.3(a)	Variation of rotational factor, r with specimen thickness in SEN bending specimens	154
7.3	Variation of rotational factor, r , (b) with a/W ratio in SEN bending and (c) with loading geometry	159
7.4	Effect of (a) thickness, (b) a/W ratio and (c) loading geometry on the variation of J-resistance curve	161
8.1	Variation of m_e with $CTOD_i$ in different loading geometries	166
8.2	Variation of CTOD at crack initiation in different specimens	166
8.3(a)	Variation of $CTOD_m$ with thickness in SEN B specimens of different a/W ratio	169
8.3(b)	Variation of $CTOD_m$ with a/W in SEN B specimen	169
8.4	Typical CTOD-resistance curves (SEN bending geometry) showing comparison with Wells' R-curves	172
8.5	Typical CTOD-resistance curves (SEN bending geometry) showing a/W effect	172
8.6	Effect of thickness on CTOD-resistance curves (SEN bending geometry)	174
8.7	Effect of thickness on $CTOD_R$ -curves (SEN tension geometry)	174
8.8	Effect of thickness and a/W ratio on CTOD-resistance curves in CT specimens	175
8.9	Effect of a/W on CTOD-resistance curves in SEN bending specimen	175

LIST OF FIGURES (Contd.)

Fig. No.	Description	Page
8.10	Typical CTOD-resistance curves for SEN tension specimens showing a/W effect	177
8.11	Variation of resistance curves with loading geometries	177
9.1	The elliptical cut-out in an infinite plate	188
9.2	Conformal mapping of ellipse to unit circle	188
9.3	Elastic-plastic stress distribution in plane strain bending for a notched bar	199
9.4(a)	Semi-infinite plate with elliptical notch	199
9.4(b)	Semi-infinite plate with an edge crack	199
9.5	Load-clip gauge displacement diagrams	204
9.5(c)	Comparison of experimental V_g for notch with generated V_g for crack	207
9.6	Variation of plastic zone (R/a) with K/σ_{ys}	211
9.7	Variation of tip opening displacement with K/σ_{ys}	215

NOMENCLATURE

a, a_{eff}	Original crack length, effective crack length
α, β	Semi-major, semi-minor axes of elliptical notch
$\delta, CTOD$	Crack-tip opening displacement
$V, CMOD$	Mouth opening displacement
$\delta_i, CTOD_i$	CTOD at crack initiation
$\delta_m, CTOD_m$	CTOD at maximum load
Δ	COD at the common point of composite crack profile
E	Young's modulus of elasticity
η	Constant of J and work done relationship
$\epsilon_x, \epsilon_y, \epsilon_z$	Normal strain in X, Y, and Z directions
G	Strain energy release rate
\bar{G}	Non-linear energy parameter
γ	Non-dimensional clip gauge opening
J, J_{Ic}	J-integral, critical plane strain J value
J_i, J_m	J-integral, at crack initiation and at maximum load
K, K_{eff}	Stress intensity factor, K based on a_{eff}
K_i, K_{Ic}	K at crack initiation and critical plane strain K value
$K(\rho), K_J$	K in finite tip radius, K from J-integral
K_σ	Stress concentration factor
m, m', M	Constraint factors to yielding
m_e	Effective plastic constraint
$V(\rho), NMOD$	Notch mouth opening displacement
NTOD	Notch tip opening displacement

ν	Poisson's ratio
n	Strain hardening exponent
ω	Flank angle
P, P_L, P_{GY}	Load, limit and general yielding load
q, q_p	Load line displacement, plastic component of q
r, r_p	Rotational factor, plastic rotational factor
r_y	Plastic zone correction factor
R, R_{max}, R_e	Plastic zone-maximum size and effective size
ρ	Notch root radius
$\sigma, \sigma_x, \sigma_y$	Normal stresses
σ_{xy}, τ	Shear stress
$\sigma_{ys}, \sigma_{flow}, \sigma_{ULT}$	Yield strength, flow stress and tensile strength
S	Span length in bending
τ_{ys}	Yield strength in shear
u, v	Displacements along X and Y directions
U, U_e, U_p	Area under P-q diagram, its elastic and plastic parts
V_g, V_{gp}	Gauge opening at distance Z from notch mouth, plastic V_g .
$V(a/W)$	Correction factor in CMOD for finite geometry
W	Width of specimen
$w(\xi)$	Mapping function in ξ -plane.

CONTENTS

		Page
	ABSTRACT ..	iii
	LIST OF TABLES ..	vii
	LIST OF FIGURES ..	viii
	NOMENCLATURE ..	xiii
CHAPTER- I	INTRODUCTION ..	1
CHAPTER-II	REVIEW OF LITERATURE ..	8
2.1	Elastic-plastic fracture mechanics parameters ..	8
2.1.1	Stress intensity factor, K ..	9
2.1.2	The Rice's J-integral ..	13
2.1.3	Crack tip opening displacement (CTOD) ..	18
2.2	Models for evaluation of CTOD ..	22
2.2.1	Location for the evaluation of CTOD ..	22
2.2.2	Experimental determination of CTOD ..	23
2.2.3	Rotational factor, r ..	26
2.2.4	Unstable fracture : (CTOD) _c ..	27
2.3	Relationship of CTOD with K and J ..	28
2.3.1	K-CTOD relation ..	28
2.3.2	J-CTOD relation ..	29
2.4	Crack growth resistance (R) ..	30
2.4.1	The R-curve ..	31
2.4.2	δ_R and J_R curves ..	32

CONTENTS (Contd.)

		Page
2.5	Aim and scope of present investigation ..	35
2.5.1	Problems with the evaluation of CTOD ..	35
2.5.2	Scope of the present investigation ..	38
CHAPTER-III	THEORETICAL FORMULATION ..	42
3.1	A mathematical model for determination of CTOD ..	42
3.1.1	Muskhelishvili solution of displacements ..	42
3.1.2	Crack opening displacement (COD) and crack mouth opening displacement (CMOD) ..	44
3.1.3	The COD and CMOD in presence of small plastic zone at the crack tip ..	47
3.1.4	The crack tip opening displacement (CTOD) model ..	49
3.1.5	The CTOD model for slow crack growth situations ..	57
3.2	Crack tip plastic zone ..	58
3.3	The physical crack expansion, Δa ..	63
3.4	Estimation of the rotational factor, r ..	64
CHAPTER-IV	EXPERIMENTAL DETAILS ..	66
4.1	Material ..	66
4.2	Fracture mechanics test specimens ..	66
4.3	Specimen preparation ..	68

CONTENTS (Contd.)

	Page
4.4 Tensile test ..	71
4.5 Fracture mechanics test set-up ..	72
4.6 Crack growth measurement ..	79
4.7 Plastic-zone measurement ..	79
4.8 Grid technique for strain measurement ..	80
4.9 Fracture test procedure ..	81
CHAPTER- V SMALL SCALE YIELDING: RESULTS ..	83
5.1 Small scale yielding defined ..	83
5.2 Load displacement diagrams ..	84
5.2.1 Load-CMOD plots ..	84
5.2.2 Load-load line displacement (P-q) diagrams ..	84
5.3 Effective crack length and plastic zone ..	91
5.4 Crack tip opening displacement ..	95
5.5 Notch-root contraction ..	97
5.6 Rotational factor, r ..	103
5.7 Evaluation of strain distribution in the vicinity of crack tip ..	105
5.8 Evaluation of J-integral ..	109
CHAPTER-VI SMALL SCALE YIELDING.: DISCUSSION ..	111
6.1 Limit of small scale yielding ..	111
6.2 Effective plastic constraint, m_e ..	114
6.2.1 Effect of specimen thickness on m_e ..	115
6.2.2 Effect of a/W ratio on m_e ..	120

CONTENTS (Contd.)

	Page
6.2.3	Effect of loading geometry on m_e .. 120
6.3	Crack tip opening displacement (CTOD) .. 123
6.3.1	Comparison of CTOD values .. 124
6.3.2	Effect of specimen thickness on $CTOD_i$.. 127
6.3.3	Effect of a/W ratio on $CTOD_i$.. 129
6.3.4	Comparison of $CTOD_i$ in different loading geometries .. 130
6.4	K-CTOD relation .. 130
6.5	Notch-root contraction (NRC) .. 138
6.6	Rotational factor, r .. 139
6.6.1	Comparison of rotational factor values .. 140
6.6.2	Rotational factor at the crack initiation point .. 142
6.6.3	Effect of yield strength on rotational factor .. 144
6.7	J-integral .. 144
6.7.1	J-integral compared with K .. 146
6.7.2	J-CTOD relation .. 146
CHAPTER-VII	ELASTIC-PLASTIC CRACK GROWTH: RESULTS .. 150
7.1	Load-displacement diagrams .. 150
7.1.1	Load versus CMOD diagrams .. 150
7.1.2	Load-crack growth (P- Δa) diagram .. 153
7.1.3	Load-load line displacement (P-q) diagram .. 156
7.2	Crack tip opening displacement .. 156

CONTENTS (Contd.)

	Page
7.3 Elastic-plastic crack growth ..	157
7.4 Rotational factor ..	157
7.5 J-resistance curves ..	158
CHAPTER-VIII ELASTIC PLASTIC CRACK GROWTH: DISCUSSION ..	162
8.1 General yielding load (P_{GY}) ..	162
8.2 Crack initiation and maximum load points ..	163
8.2.1 CTOD at the crack initiation point ..	163
8.2.2 CTOD at the maximum load point, $CTOD_m$..	168
8.3 CTOD crack-resistance curve ..	170
8.3.1 Comparison between the crack- resistance curves ..	171
8.3.2 Effect of Thickness on $CTOD_R$ - curve ..	173
8.3.3 Effect of a/W ratio on δ_R - curve ..	173
8.3.4 Effect of loading geometry on δ_R -curve ..	178
8.4 J-resistance curves ..	178
8.5 Rotational factor (r) in slow stable crack growth ..	184
8.5.1 Effect of specimen thickness (B) on r ..	184
8.5.2 Effect of a/W ratio on r ..	185
8.5.3 Effect of loading geometry on r ..	185

CONTENTS (Contd.)

	Page
CHAPTER-IX	SOME APPLICATIONS TO NOTCHED PLATES .. 187
9.1	Tip opening displacement in notched bars .. 187
9.1.1	Derivation for mouth opening displacement .. 187
9.1.2	Notch tip opening displacement (NTOD) .. 195
9.2	Plastic zone in notched specimens .. 196
9.2.1	Plastic zone size in a SEN tension specimen .. 196
9.2.2	Plastic zone size in SEN bending specimen .. 197
9.3	Stress intensity factor in a notched plate .. 201
9.4	The experimental results .. 203
9.5	Variation of plastic zone size, R with K/σ_{ys} .. 208
9.5.1	Elastic stress concentration factor .. 208
9.5.2	Plastic zone size in SEN bending specimen .. 209
9.5.3	Plastic zone size in SEN tension specimen .. 210
9.5.4	Comparison of plastic zone size in different specimens .. 210
9.6	Variation of notch tip opening displacement .. 212
CHAPTER-X	CONCLUSIONS .. 217

CONTENTS (Contd.)

			Page
APPENDIX- I 225
APPENDIX- II 226
APPENDIX-III 227
APPENDIX- IV 232
APPENDIX- V 235
REFERENCES 236