

FINITE ELEMENT STUDY OF MULTI-SIZE PARTICULATE FLOW THROUGH ROTATING CHANNEL

by

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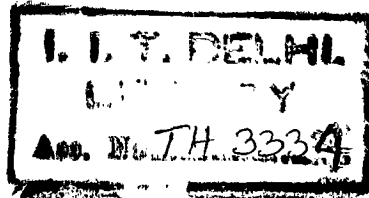
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CERTIFICATE

This is to certify that the thesis entitled **FINITE ELEMENT STUDY OF MULTI-SIZE PARTICULATE FLOW THROUGH ROTATING CHANNEL** being submitted by **Pankaj Kumar Gupta** to the **Indian Institute of Technology, New Delhi (India)**, for the award of the degree of **Doctor of Philosophy in Applied Mechanics Department** is a bona fide work carried out by him under my supervision and guidance. The research reports and the results presented in this thesis have not been submitted in parts or in full to any other University or Institute for the award of any other degree or diploma.



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ABSTRACT

The present study deals with finite element study of dense multi-size particulate flow through a rotating channel. The broad particle size distribution – common in industrial applications – is represented by discrete size classes (termed as species), each with a representative particle diameter. The volume-averaged equations governing two-dimensional multi-size particulate flow in a rotating frame of reference is described mathematically using the continuum-mechanical model. Thus for each species, continuity and momentum equations are written. The species momentum equations include lift, drag and virtual mass forces. In addition to the species equations, the momentum and continuity equations for the mixture (treated as a pseudofluid) are also considered. Eulerian boundary conditions are specified for the mixture and species equations. Turbulence is modeled using three different $k-\varepsilon$ models, all of which incorporate a rotation modification to the production term in the turbulent kinetic energy equation. In the first model, rotation-modified $k-\varepsilon$ equations are solved for the pure carrier. The resultant eddy viscosity is then modified for the presence of particles. In the other two models, rotation-modified $k_m - \varepsilon_m$ equations are solved for the mixture directly. The eddy viscosities are then computed from k_m and ε_m .

Galerkin finite element methodology (GFEM) with Q_1Q_0 elements is employed to obtain the weak form of the governing equations. The mixture flow field, solid species velocity and concentration fields are solved in a sequential approach. Separate combined Newton's iteration is used to obtain solution for each of the pure carrier flow field, mixture flow field, solid velocity field and concentration fields for each species and $k-\varepsilon$ fields for pure carrier-phase and mixture.

Due to the heavy computational demand, a multifrontal direct solver (UMFPACK 4.4) is employed to solve the large system of algebraic equations. To further increase the computational efficiency, an iterative GMRES solver is also

developed. The multifrontal solver is used to obtain a suitable preconditioner for the GMRES solver.

The predicted results of all three models show very good mesh independence. The coarsest mesh satisfied the mass conservation to within 0.1%. The code based on pure carrier-phase eddy viscosity (modified for concentration) is computationally 2-3 times more efficient than the other two (mixture-based) models. GMRES solver (with preconditioner obtained from multifrontal direct solver) is found to take 30% - 60% (depending on the mesh) of the total computation time as that of the multifrontal direct solver.

The predicted results using all three turbulence models are compared against applicable experimental results in the open literature. Regression between the computed and experimental results shows excellent agreement between the two. The predicted results using the three turbulence models over a wide range of operating parameters showed respectable agreement; results of all the three models agree within about 11% (RMS difference) in overall concentration and about 6% (RMS difference) in mixture velocity.

Results indicate a multifaceted relationship among the Coriolis and centrifugal forces and the effects of turbulence. Sensitivity of the results to the modeled terms and parameters is investigated. The extent of rotation modification and centrifugal force tend to have a strong effect on the solution. On the other hand, the exclusion of lift force, virtual mass force (in the species momentum equations) and the diffusion stress terms (in the mixture momentum equations) seem to have negligible influence on the solution for the range of parameters studied. Likewise the wall function parameter is found to not have a significant effect. In general (within the range of operating parameters covered in the present study), with increased rotational speed and increased inlet (average) concentration, mixture velocity increases (decreases) near the pressure (suction) side and the concentration differential (difference in

concentration at pressure and suction sides) increases. Larger size particles experience more Coriolis force compared to smaller size particles. Consequently PSD's which have more concentration of the larger size particles tend to have more concentration differential along the channel cross section. Similar effects results with increased bulk flow Reynolds number for a channel of fixed geometry and fluid at fixed rotation.

A direct bearing of the predicted flow field is on erosion wear. For a specific slurry/wear material combination, wear rates at the pressure side are higher than at the suction side. The general trends are that the maximum wear rate usually occurs either near the inlet or near the exit subject to the operating parameters. With increase in rotation speed, the increase in local solids concentration differential increases the wear rate on the pressure side and a corresponding decrease on the suction side. An increase in the bulk Reynolds number and the average inlet concentration also produce a similar effect on the wear rates.

Key words : Multi-size parti-

Galerkin fin

Turbulent flow

Rotating channel

Finite element method

channel, turbulence modification,

Coriolis and centrifugal forces

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