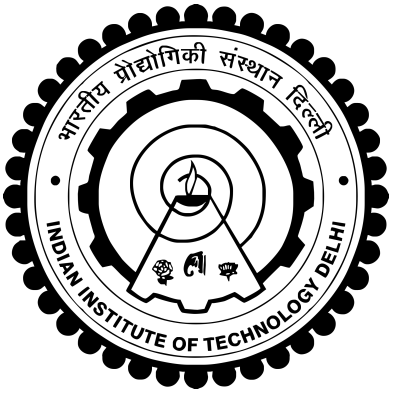
MANEUVERING OF SERPENTINE ROBOTS: AN ADAPTIVE ROBUST CONTROL APPROACH

JOYJIT MUKHERJEE



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MANEUVERING OF SERPENTINE ROBOTS: AN ADAPTIVE ROBUST CONTROL APPROACH

by

JOYJIT MUKHERJEE

Department of Electrical Engineering

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विद्या ददाति विनयं विनयधाति पत्रताम् ।
पात्रत्वाद्धनमाप्नोति धनाद्धर्मं ततः सुखम् ॥

- हितोपदेशः

Knowledge gives humility; from humility one attains character;
from character, one acquires wealth; from wealth follows good deeds and happiness.
- Hitopadesha

To Thakuma, Baba, Ma and Didi ...

Certificate

This is to certify that the thesis entitled **"Maneuvering Of Serpentine Robots: An Adaptive Robust Control Approach"**, submitted by **Joyjit Mukherjee** to the Indian Institute of Technology Delhi, for the award of the degree of **Doctor of Philosophy** in Electrical Engineering, is a record of the original, bona fide research work carried out by him under our supervision and guidance. The thesis has reached the standards fulfilling the requirements of the regulations related to the award of the degree.

The results contained in this thesis have not been submitted in part or in full to any other University or Institute for the award of any degree or diploma to the best of our knowledge.

Prof. Indra Narayan Kar

Department of Electrical Engineering,

Indian Institute of Technology Delhi.

Prof. Sudipto Mukherjee

Department of Mechanical Engineering,

Indian Institute of Technology Delhi.

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Joyjit Mukherjee

Abstract

Snake robots are articulated systems designed to imitate the motion characteristics of biological snakes. These systems find range of prospective applications due to their salient features like multi-modal locomotion and flexible motion capabilities. However, these features also pose challenges from the point of view of dynamical modeling and control. Hence, research in this field have find traction to make these systems worthy of field deployment.

The motion of a snake robot on a flat surface has been widely studied in literature. The state-of-art control approach for the planar snake robot, employs a VHC based body shape control along with singular perturbation based head-angle control and feedback linearization based velocity control. This approach requires the system to be known, which can be considered as a conservative assumption for unstructured and unknown environments. Moreover, the complicated dynamical system structure makes the control design problem challenging which has been conventionally solved through a multi-layer approach. Furthermore, to incorporate the versatility of snake motion in the robot, it is necessary to investigate the other modes of locomotion as well. Such studies would enhance the applicability of these systems in real-life. The major contributions of this thesis addressing the aforementioned issues are detailed below:

- A sliding mode control (SMC) approach has been adopted to achieve efficient tracking for a planar snake robot considering bounded uncertainties in the friction coefficients with known upper bound. To circumvent the requirement of knowledge of the upper bound, an adaptive-sliding mode control (ASMC) law has been devised, which also alleviates the

overestimation of the switching gain.

- The SMC and ASMC methodologies require the uncertainties to be bounded and are also dependent upon the friction model. A time-delayed control (TDC) approach utilizing an artificial delay based estimation, has been adopted to obtain satisfactory tracking efficiency of planar snake robot while obviating the aforementioned assumptions.

- The bounded estimation error in TDC has a detrimental effect on the tracking performance of a planar snake robot, which can only be reduced by choosing high feedback gain. For this reason, an adaptive robust time-delayed control (ARTDC) law has been devised to achieve improved tracking performance in the presence of bounded estimation error. The novel dual-rate adaptation law is aimed at solving the overestimation problem of the switching gain.

- Differential flatness has been studied and employed for trajectory tracking of wheeled mobile robot. Subsequently, flatness has been used to establish mapping between the output and the input variables of the robot utilizing the tangential position as the flat output. Finally, the serpenoid gait function has been used to derive a flat system for a planar snake robot, which has been further utilized for controller design.

- The dynamic equations of motion of a snake robot inside a channel has been obtained through Newton-Euler formulation. The normal and traction forces generated through the interaction of robot link and channel wall have been modeled. A control law has been employed to study the tracking performance of the robot inside a uniform channel.

Overall, this thesis contribute towards the dynamic modeling and control design of snake robots in an attempt to enhance their deployability.

सारांश

स्नेक रोबोट जैविक साँपों की गति विशेषताओं की नकल करने के लिए डिज़ाइन किए गए सिस्टम हैं। ये सिस्टम बहु-मोडल लोकोमोशन और लचीली मोशन क्षमताओं जैसी अपनी मुख्य विशेषताओं के कारण संभावित परिदृश्यो मे इस्तेमाल किए जाते हैं। हालाँकि, ये सुविधाएँ गतिशील मॉडलिंग और नियंत्रण के दृष्टिकोण से चुनौतियों का सामना करती हैं। इसलिए, इन प्रणालियो को क्षेत्र परिनियोजन की योग्य बनाने के उद्देश्य से इस क्षेत्र में अनुसंधान महत्वपूर्ण है।

एक सपाट सतह पर रोबोटिक सांप की गति को साहित्य में व्यापक रूप से अध्ययन किया गया है। प्लानर स्नेक रोबोट के लिए अत्याधुनिक नियंत्रण दृष्टिकोण, एक वीएचसी आधारित बॉडी शेप नियंत्रण के साथ-साथ सिन्गुलर पर्टबेशन पर आधारित हेड-एंगल नियंत्रण और फीडबैक लीनियराइजेशन आधारित वेग नियंत्रण को नियोजित करता है। इस विधि मे सम्पूर्ण सिस्टम की जानकारी आवश्यकता है, जिसे असंरचित और अज्ञात वातावरण के लिए एक रूढ़िवादी धारणा के रूप में माना जा सकता है। इसके अलावा, जटिल गतिशील प्रणाली संरचना के लिए नियंत्रण डिजाइन की समस्या को पारंपरिक रूप से बहु-परत दृष्टिकोण के माध्यम से हल किया गया है। इसके साथ ही, रोबोट मे साँप की गति की बहुविधता को शामिल करने के लिए, चलन के अन्य तरीकों की भी जांच करना आवश्यक है। इस तरह के अध्ययन वास्तविक जीवन परिदृश्यों में इन प्रणालियों की प्रयोज्यता को बढ़ाएंगे। उपर्युक्त मुद्दों को संबोधित करने वाले इस थीसिस के प्रमुख योगदान निम्नलिख हैं:

- ज्ञात ऊपरी सीमा के साथ घर्षण गुणांक में सीमित अनिश्चितताओं पर विचार करते हुए, एक प्लैनर स्नेक रोबोट की कुशल ट्रैकिंग प्राप्त करने के लिए एक स्लाइडिंग मोड कंट्रोल (एस.एम.सी) दृष्टिकोण अपनाया गया है। ऊपरी सीमा के ज्ञान की आवश्यकता को दरकिनार करने के लिए, एक एडाप्टिव-स्लाइडिंग मोड कंट्रोल (ए.एस.एम.सी) विधि तैयार किया गया है, जो स्विचिंग गेन की अधिकता को भी कम करता है।

- एस.एम.सी और ए.एस.एम.सी कार्यप्रणालियों की लिए सीमित अनिश्चितताओं की आवश्यकता होती है और यह घर्षण मॉडल पर भी निर्भर करता हैं। कृत्रिम विलंब आधारित अनुमान का उपयोग करने में टाईम डिलेड़ कंट्रोल (टी.डी.सी) दृष्टिकोण, उक्त मान्यताओं का दरकिनार करते हुए, प्लैनर स्नेक रोबोट की संतोषजनक ट्रैकिंग दक्षता प्राप्त करने के लिए अपनाया गया है।

- टी.डी.सी की सीमित अनुमान त्रुटि का एक प्लैनर स्नेक रोबोट के ट्रैकिंग प्रदर्शन पर हानिकारक प्रभाव पड़ता है, जिसे केवल उच्च फ़ीडबैक गेन चुनकर कम किया जा सकता है। इस कारण से, एडाप्टिव रोबस्ट टाईम डिलेड़ कंट्रोल (ए.आर.टी.डी.सी) प्रणाली तैयार की गई है जो कि सीमित अनुमान त्रुटि की उपस्थिति में बेहतर ट्रैकिंग प्रदर्शन करती है। एक नई दोहरे दर अडपटेशन विधि, स्विचिंग गेन की अति-आकलन समस्या को हल करने के उद्देश्य से है।

- व्हील्ड मोबाइल रोबोट के प्रक्षेपवक्र ट्रैकिंग के लिए डिफरेंशियल फ्लैटनेस का अध्ययन और नियोजन किया गया है। इसके बाद, टैंनजेंशिएल पोजिशन को फ्लैट आउटपुट मानते हुए सिस्टम की आउटपुट और इनपुट के बीच मैपिंग को स्थापित करने के लिए फ्लैटनेस का उपयोग किया गया है। अंत में, सेरपेनॉइड गेट फ़ंक्शन का उपयोग करते हुए एक प्लैनर स्नेक रोबोट के लिए फ्लैट सिस्टम प्राप्त किया गया है, जिसका उपयोग नियंत्रक डिजाइन के लिए आगे किया गया है।

- एक नली के अंदर रोबोटिक सांप की गतिशील समीकरण को न्यूटन-ऑयलर फॉर्मूलेशन के माध्यम से प्राप्त किए गए हैं। रोबोट लिंक और नली की दीवार के बीच उत्पन्न संपर्क और कर्षण बलों को मॉडल किया गया है। एक समान नली के अंदर रोबोट के ट्रैकिंग प्रदर्शन का अध्ययन करने के लिए एक नियंत्रण विधि नियुक्त की गई है।

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Abbreviations

Abbreviation/Symbol	Description
CM	Centre of Mass
BSC	Body Shape Control
VHC	Virtual Holonomic Constraints
SMC	Sliding Mode Control
ASMC	Adaptive Sliding Mode Control
TDC	Time-Delayed Control
TDE	Time-Delayed Estimation
ARC	Adaptive Robust Control
ARTDC	Adaptive Robust Time-Delayed Control
UUB	Uniformly Ultimately Bounded
WMR	Wheeled Mobile Robot

Nomenclature

Symbol	Symbol Description	Corresponding Vector
$\mathbb{R}$	Set of Real Numbers	
$\mathbb{R}^+$	Set of Positive Real Numbers	
$\mathbb{R}^j$	Real space of dimension j	
$\mathbb{R}^{j\times j}$	Real matrix of dimension $(j \times j)$	
$\exists$	there exists	
$\forall$	for all	
$\mathbf{I}_{j\times j}$	Identity matrix of dimension $(j \times j)$	
$\mathbf{0}_j$	Zero vector of dimension j	
$\mathbf{0}_{j\times j}$	Zero matrix of dimension $(j \times j)$	
$\text{rand}\,(j)$	Vector of dimension j with random values within [0 1]	
$\mathbf{\Lambda} > \mathbf{0} (< \mathbf{0})$	Positive (Negative) definite matrix	
$\lambda_{\min}(\mathbf{\Lambda})$	Minimum eigenvalue of matrix $\mathbf{\Lambda}$	
$\lambda_{\max}(\mathbf{\Lambda})$	Maximum eigenvalue of matrix $\mathbf{\Lambda}$	
$\|\mathbf{\Lambda}\|$	Euclidean norm of matrix $\mathbf{\Lambda}$	
n	Number of links for snake robot	
c_t	Tangential ground friction coefficient	
c_n	Normal ground friction coefficient	
m	Mass of each link	
$2l$	Length of each link	
J	Moment of inertia of each link	
θ_i	Angle of ith link with respect to global X axis	$\boldsymbol{\theta} \in \mathbb{R}^n$
(p_x, p_y)	Position of the robot CM in global reference frame	$\mathbf{p} \in \mathbb{R}^2$
(x_i, y_i)	Position of the CM of ith link in global reference frame	$\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$
τ_i	Torque applied on ith link by $(i+1)$th link	$\boldsymbol{\tau} \in \mathbb{R}^{n-1}$
a	Semi-minor axis of elliptical link for in channel motion	
b	Semi-major axis of elliptical link for in channel motion	
d	Width of the channel	
$(x_{i,min}, y_{x_{i,min}})$	Point on ith link with minimum abscissa	
$(x_{i,max}, y_{x_{i,max}})$	Point on ith link with maximum abscissa	
E_{rub}	Modulus of elasticity of link surface material	
ν_{rub}	Poisson's ratio of link surface material	
ϵ_{rub}	Maximum virtual penetration of Link surface	

Symbol	Symbol Description	Corresponding Vector
$N_{i,l}$	Contact force applied by the left wall on ith link	$\mathbf{N}_l \in \mathbb{R}^n$
$N_{i,r}$	Contact force applied by the right wall on ith link	$\boldsymbol{N}_r \in \mathbb{R}^n$
$C_{f,c,i}$	Total Contact force on ith link along X direction	$\mathbf{C}_{f,c} \in \mathbb{R}^n$
$v_{i,l}$	Contact point velocity of ith link and left wall	
$v_{i,r}$	Contact point velocity of ith link and right wall	
$r_{i,l}$	Distance of ith link CM and its left wall contact point	
$r_{i,r}$	Distance of ith link CM and its right wall contact point	
$\mu_{i,l}$	Friction coefficient for ith link left wall contact	
$\mu_{i,r}$	Friction coefficient for ith link right wall contact	
$C_{i,l}$	Friction force applied by the left wall on ith link	$\mathbf{C}_l \in \mathbb{R}^n$
$C_{i,r}$	Friction force applied by the right wall on ith link	$\mathbf{C}_r \in \mathbb{R}^n$
$C_{f,y,i}$	Traction force on ith link along Y direction	$\mathbf{C}_{f,y} \in \mathbb{R}^n$
$C_{f,\tau,i}$	Moment by Contact and Traction force on ith link	$\mathbf{C}_{f,\tau} \in \mathbb{R}^n$
$h_{x,i}$	Joint constraint force from ith link to $(i+1)$th link along global X direction	$\mathbf{h}_x \in \mathbb{R}^{n-1}$
$h_{y,i}$	Joint constraint force from ith link to $(i+1)$th link along global Y direction	$\mathbf{h}_y \in \mathbb{R}^{n-1}$