SOLUTION OF INVERSE PROBLEM IN GROUNDWATER MODELLING

by

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Certificate

This is to certify that the thesis entitled 'Solution of Inverse Problem in Groundwater Modelling' being submitted by Sushil Kumar Goyal , to the Indian Institute of Technology, Delhi for the award of the degree of Doctor of Philosophy, is a record of bonafide research work carried out by him under our guidance and supervision. In our opinion, the thesis has reached the requisite standard fulfilling the requirements of the regulation relating to the said degree. The material contained in this thesis has not been submitted, in part or full, to any other university or institute for the award of any degree or diploma.

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SYNOPSIS

Introduction

A proper understanding of the aquifer behaviour to pumping and recharge stresses is necessary for quantitative evaluation of groundwater resources. For this purpose a model representing the flow through an aquifer is chosen. The most commonly used model is the deterministic physically based model described by a second order parabolic partial differential equation. In order to make this model operational, information about the aquifer parameters is necessary. These parameters are determined by analysis of head variations in a pump-test or natural groundwater level fluctuations. The problem of finding out the aquifer parameters from the known heads is termed as the inverse problem in groundwater modelling.

Existing methods for analysis of pump-test data

Conventional analysis of aquifer test data is carried out by graphical techniques which employ analytical solutions of the governing partial differential equations under simplified conditions. For nonleaky aquifers, the transmissivity and storage coefficient are estimated by fitting the experimental data to a graphic analogue of the Theis equation. For leaky aquifers, the Hantush and Jacob (1955) solution is commonly used to get the estimates for transmissivity, storage coefficient and leakage factor by graphical methods.

The graphical methods involve considerable judgement on the part of the analyst and do not give a quantitative measure of the accuracy of matching the test data or an index of reliability of the parameter estimates. Other attempts such as quasilinearisation and discrete numerical models for analysing the pump-test data involve convergence problems or considerable formulation and computational effort.

Most often, the field observations of head variations are susceptible to errors. In addition, there are system uncertainities due to the assumptions made in deriving the governing equations and in getting their closed form solutions. The existing techniques for analysing the pump-test data cannot take into account these uncertainities in the system and in the observations.

Existing methods for analysis of natural groundwater levels

Indirect and direct methods are used to solve the governing partial differential equations for analysing the ground water level fluctuations. Indirect methods carry out iterative adjustment of the initial parameter estimates in a mathematical programming problem until a satisfactory agreement is reached between the observed and the calculated water levels. These methods require repetitive solutions of the flow equation and thus involve a considerable computational effort. Direct methods treat the parameters as dependent variables in the flow equation and solve for them directly. The problem is reduced to that of solving a set of algebraic equations or a formal boundary value problem. These methods do not require repetitive solutions of the flow equation, but the inherent instability in aquifer identification is likely to be more pronounced in the direct methods (more so, in the presence of system and observational errors) which may lead to physically unrealistic estimates. Direct methods attempted for transient flow analysis such as energy dissipation, linear programming, flatness criterion and spline interpolation yield physically unrealistic estimates or involve considerable computational effort and require a large amount of data.

Theme of the present study

The theme of the present study is to overcome some of the shortcomings in the existing techniques for the solution of the inverse problem in groundwater modelling. This has been achieved through suitable modifications of the flow equation and by using Marquardt algorithm (Marquardt, 1963) and iterated extended Kalman filter (Jazwinski, 1970) techniques in identifying aquifer parameters from the pump-test data. These techniques also give an index of reliability of the parameter estimates. Recursive least-squares (Goodwin and Pyne, 1977) and stochastic approximation (Kubrusly, 1978) techniques are used to obtain estimates of diffusivity distribution in an aquifer from natural groundwater levels with very little computational effort as compared to the existing methods. The estimates are realistic. These techniques take into account the system and measurement uncertainties.

Proposed methods for analysis of pump-test data

As mentioned above, Marquardt algorithm and iterated extended Kalman filter techniques have been employed for analysing the pump test data. The Theis equation for non-leaky aquifers and Hantush and Jacob solution for the leaky aquifers have been suitably modified for application of these techniques.

Marquardt algorithm acts as an open loop controller and carries out least-squares estimation of nonlinear aquifer parameters. The initial estimates are iteratively upgraded till a close agreement is reached between the calculated and

and observed drawdowns during some history period. The algorithm is applied to actual pump-tests in nonleaky confined, unconfined and semi-confined aquifers. The algorithm shows quick convergence. The results of the analysis are compared with those given by the graphical methods, quasi-linearisation and numerical models. It is found that the standard error of matching between the observed drawdowns and those calculated using the estimated set of parameters, is the least for the proposed method.

An iterated extended Kalman filter acts as a feedback controller and filters out the effect of noise in modelling and observations to provide the minimum variance estimates of the parameters. The model output is continuously assessed against the actual system output and the residual error between the actual and the predicted output is used as a feedback information into the parameter adjustment mechanism. Thus the filter updates the parameter estimates with each additional information and also provides their estimation error covariance. The filter is applied to actual pump-test data for confined nonleaky and leaky aquifers. The filter shows a good convergence. Confidence limits on the parameter estimates are established from the estimation error covariance matrix. Results of the analysis are compared with those given by graphical and numerical methods. The standard error of matching is the minimum for the proposed filter.

The Marquardt algorithm and the iterated extended Kalman filter eliminate the subjectivity inherent in the graphical methods. The suggested methods ensure good convergence and are simple in formulations. The Marquardt algorithm gives the standard error of matching the field data. The iterated extended Kalman filter takes into account the uncertainties and also gives the confidence limits of the parameter estimates.

Proposed methods for analysis of groundwater level fluctuations

Realistic estimates of diffusivity distribution in an aquifer from natural groundwater levels are determined by 'recursive least squares' and 'stochastic approximation' using the suitably modified forms of the governing partial differential equations for groundwater flow. The techniques are conceptually and computationally simple and are also sequential since the parameters are updated with each additional set of observations.

Recursive least-squares technique is the recursive version of the classical least-squares method and it avoids the expanding memory and time requirement. The introduction of a regularisation criterion in the form of initial diffusivity estimates and covariance controls the non-unicity of the solution. An implicit centra-difference scheme is used to discretise the governing equation for the radial as well as the general two-dimensional flow and then the recursive least-squares method is employed to determine the diffusivity estimates. The method is applied to recover the diffusivity estimates from the simulated data of a nonleaky, confined, radially heterogeneous reservoir. The estimates are found to converge quickly. The technique is applied also to get the diffusivity distribution from the simulated data for two-dimensional groundwater flow in a rectangular reservoir.

The stochastic approximation technique is an extension of the gradient methods to stochastic systems and is used when the statistics of observational noise are known. It involves the recursive estimation of only the second order moments of the noisy observation process. The governing partial differential equation for radial flow is discretised using forward and central explicit finite differences and the observation dynamics is described in terms of the previous drawdowns and the noise in modelling and observations. Stochastic approximation technique is used to determine the second-order moments of the observation process recursively which in turn provide the diffusivity estimates. Convergence is assured by proper selection of two scalar sequences. The technique is applied to recover diffusivity estimates from the data simulated for a radially heterogeneous reservoir. The diffusivity estimates tend to converge quickly. The stochastic approximation technique requires water level observations at short time intervals so as to satisfy the stability criterion of the explicit scheme. However, it takes into account the uncertainties and is computationally simpler than the recursive least-squares method. Both these techniques provide realistic estimates of diffusivity distribution in an aquifer system and are shown to be computationally simple.

To sum up, the flow equations have been suitably modified to enable the use of the above mentioned techniques in determining the aquifer parameters, taking into account the errors in the system as well as in observations. The study reveals that these formulations are robust, require less computational effort and yield realistic estimates.

TABLE OF CONTENTS

Page