RECONSTRUCTION BASED RECOGNITION OF REPEATED 3D OBJECTS USING INVARIANTS

by

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Certificate

This is to certify that the thesis entitled "**Reconstruction Based Recognition of Repeated 3D Objects using Invariants**," which is being submitted by Ms. Ragini Choudhury to the Department of Mathematics, Indian Institute of Technology, Delhi, for the award of the degree of Doctor of Philosophy, is a record of bonafide research work, carried out by her under our guidance and supervision. The thesis has reached the standards fulfilling the requirements of the regulations relating to the degree. The results contained in this thesis have not been submitted to any other university or institute for the award of any degree or diploma.

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Ragini Choudhury

Abstract

In this thesis, we have considered the problem of

- Recognition of 3D configurations with repeated (two or more) components, using a single uncalibrated perspective image of such a configuration. The three types of repetition we have considered are
  - Translational repetition
  - Affine repetition
  - Euclidean repetition
- Repeated components have been considered to be quadrics, as a specific case

The proposed recognition scheme involves developing an image computable projective reconstruction of the components from a single image and computation of projective invariants of these repeated components.

Our process of invariant computation is reconstruction based and the reconstructions are obtained using the relative affine structure and suitable homographies. A new mathematical framework using relative affine structure has been proposed. Using this the repeated objects can be reconstructed projectively, with image correspondences of any four identifiable points, in general position, on the object and its repetition. This is extended to obtain a reconstruction of repeated quadrics with the additional image information of outline conics. The relative affine structure has been necessitated by quadric reconstruction. Joint projective invariants of a set (two/three) of quadrics

have been proposed. For the purpose of recognition of quadric configurations, these invariants are computed using the reconstructed quadrics.

The projective reconstruction (relative affine structure) matrix is such that it makes the reconstruction image computable in case of translationally repeated objects. In the case of configurations with two repeated components, when considering affine repetition, we have shown that the reconstruction is image computable if the infinite homography H_∞ can be computed. For the case of rigid repetition, we have proposed a new Linear Programming based method for computing infinite homography, which makes H_∞ computable from image information. Using this, an image computable reconstruction of repeated quadrics is given. We have computed joint projective invariants of a pair of proper quadrics, applicable to all quadric pairs, irrespective of whether they are repeated or not. These are used to compute invariants of configurations with two repeated quadric components. The recognition strategies have been illustrated via application on real life objects and scenes. The invariants have been found to possess a good discriminatory ability.

We have also proposed an invariance based recognition scheme for scenes with multiple translationally repeated components, using a single view of the scene. This is done by identifying three component subsets and projectively reconstructing each such subset, using the mathematical framework developed by us. This is extended to the reconstruction when each component is a quadric. We have computed a set of joint projective invariants for three proper quadrics. These are evaluated for each quadric component subset and the values are used to recognize the scene as a whole. We have experimentally shown that the invariants of the set of three quadrics, have a high discriminatory ability and are stable.

The main contributions of the thesis are

- Invariance based recognition strategies for 3D configurations with repeated (two or more) components, using a single image of such a configuration. Repetition can be a translation, Euclidean or affine transformation. The strategies are reconstruction based.

- We have applied the recognition strategies only to configurations with a pair of repeated quadrics or to scenes with three or more quadric components, but the strategy is general and applicable to all repeated objects for which invariants can be computed.

- The reconstruction scheme for repeated objects is also general and applicable to different classes of repeated objects.

- The Linear Programming based method for computing the infinite homography is not specific to the type of objects and can be applied for other purposes, like self-calibration, which require computation of coordinates of the plane at infinity.

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