

**PROPAGATION OF ACCELERATION AND SHOCK  
WAVES IN PHYSICAL GAS DYNAMICS :  
ANALYTICAL AND COMPUTATIONAL STUDIES**

By  
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*... To My Parents ...*

CERTIFICATE

*This is to certify that the thesis entitled "PROPAGATION OF ACCELERATION AND SHOCK WAVES IN PHYSICAL GAS DYNAMICS: ANALYTICAL AND COMPUTATIONAL STUDIES" which is being submitted by Mrs. A.Lalita Dixit for the award of degree of Doctor of Philosophy in Mathematics to the Indian Institute of Technology, Delhi, is a record of bonafide research work carried out by her under my guidance and supervision.*

*The thesis has reached the standard fulfilling the requirements of the regulations relating to the degree. The results obtained in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.*

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## ABSTRACT

This thesis entitled "PROPAGATION OF ACCELERATION AND SHOCK WAVES IN PHYSICAL GASDYNAMICS: ANALYTICAL AND COMPUTATIONAL STUDIES" consists of five chapters. Chapter I gives a brief review of the previous studies related to the present work and a discussion concerning the set of equations governing the gas flow under study. Chapter II deals with the application of method of characteristics to study the propagation of acceleration waves in

- I        Radiation - Magneto Gasdynamics ,
- II       Radiative and Vibrational Non-equilibrium Gasdynamics ,
- III      Non-equilibrium Magneto Gasdynamics.

A differential equation governing the growth and decay of an acceleration wave is derived and integrated numerically. The critical amplitude and the critical time are also obtained. We have concluded that in Case I the effect of magnetic field on compressive waves which owe their origin to radiation is to cause an early shock formation, while on those of expansion waves, the effect of magnetic field is to decrease the decay rate. However, the coupling of radiation and magnetic field on waves which are emanating from the piston movement is to delay the cylindrical shock formation.

In Case II, the growth and decay of the acceleration wave depend on the measure of net rate of energy transfer. In this case we have also studied the effect of ambient non-equilibrium

on the propagation of acceleration wave. In Case III, we have observed that the effect of magnetic field on the steepening and flattening of acceleration wave depends on the time taken for relaxation processes.

In Chapter III, by using singular surface theory we have studied the non-uniform propagation of weak discontinuities in

- I Radiation-Magneto Gasdynamics,
- II Radiative and Vibrational Non-equilibrium Gasdynamics,
- III Thermally Conducting and Dissociating Gasdynamics.

The transport equations are formulated and solved completely. Further the criteria for shock formation have been established. It turns out that the effect of ambient non-equilibrium on the propagation of weak wave in a general relaxing gas is to amplify the disturbance. But the presence of ambient radiative non-equilibrium in Case II causes a decrease in the magnitude of disturbance. We have also shown that the coupling of thermal conduction and dissociation has a stabilizing effect so as to increase the time taken to form a shock wave. On the non-uniform propagation of weak wave in a radiation-magneto gasdynamics, the effects of magnetic field, thermal conduction, and radiation are to delay the development of a shock while the electrical conductivity causes the rapid onset of it.

In Chapter IV, by combining ray theory with the technique of compatibility conditions of Thomas, we have studied the shock

strength variation with time along the ray, and differential effects of shock in the following:

- I        Thermally Conducting and Radiative Gasdynamic flow,
- II       Radiation-Magneto Gasdynamic flow,
- III      Thermally Conducting and Dissociating Gasdynamic flow.

We have observed that in a thermally conducting gas the shock strength decreases with increase in time and finally terminates into a weak wave. Increase in the Radiation flux number in Case I and II causes to slow down the decay rate of shock strength. Special consideration has been given to parallel, perpendicular and switch-off hydromagnetic shocks. In Case III we have also obtained the velocity of a shock.

In Chapter V, by taking initial conditions as in a shock tube, we have used finite difference methods to study the propagation of converging cylindrical shock in a Radiation Gasdynamics. We have observed that the effect of radiation heat transfer is to decrease the growth rate of shock strength while it is propagating towards the axis. Further, the effect of radiation is to delay the shock coalescence with the axis.

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