

CREEP FATIGUE INTERACTION STUDY OF STEAM TURBINE ROTORS USING CONTINUUM DAMAGE MECHANICS AND COMPUTATIONALLY EFFICIENT NUMERICAL TECHNIQUES

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INDIAN INSTITUTE OF TECHNOLOGY DELHI
MARCH 2024**

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CONTINUUM DAMAGE MECHANICS AND
COMPUTATIONALLY EFFICIENT
NUMERICAL TECHNIQUES**

by

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Submitted

in fulfilment of the requirements of the degree of

DOCTOR OF PHILOSOPHY

to the



INDIAN INSTITUTE OF TECHNOLOGY DELHI

MARCH 2024

*Dedicated to
my near and dear ones
who silently endured all the pain to see this day.*

Certificate

This is to certify that the thesis entitled “**Creep Fatigue Interaction Study of Steam Turbine Rotors using Continuum Damage Mechanics and Computationally Efficient Numerical Techniques**” being submitted by **Mr. Suvadeep Sen** to the Indian Institute of Technology Delhi for the award of degree of **Doctor of Philosophy** in Applied Mechanics is a record of original, bonafide research work carried out by him under my supervision and guidance. The thesis work, in my opinion, has reached the requisite standard fulfilling the requirements for the degree of Doctor of Philosophy.

The results contained in this thesis have not been submitted in part or in full, to any other university or institute for the award of any degree or diploma.

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Acknowledgements

The cover page of this thesis deserves more names than just one. This research work would not have been possible without the continuous support, encouragement and sacrifices of several individuals.

I would like to express my gratitude to my supervisor Professor B.P. Patel, Department of Applied Mechanics, IIT Delhi for his inspiring guidance, motivation, critical analysis and detailed review of the research work. It was an honour to work under the dedicated supervision of such a committed professional and an excellent human being. I am thankful to my research committee members for their reviews and faculty members of the Department of Applied Mechanics, IIT Delhi for their kind help and support.

Pursuing research after a gap of a decade in academics definitely needed a lot of inspiration. My wife Moumita always motivated me to enrich myself with further education ever since we were married. In addition, my erstwhile manager at Siemens, Mr. Rajiv Singh Bais and Professor S.P. Singh, Department of Mechanical Engineering, IIT Delhi provided me with the launching pad and the fuel to take up the Ph.D. program. Special appreciation also goes to Mr. Sumit Kumar, Ph.D. scholar at IIT Delhi, who helped me with this transition and for having intense technical discussions on interesting topics, having a peer like him was truly inspirational.

Identifying the topic of research was the first challenging step, I am grateful to Mr. Henning Almstedt, Lead Technical Expert, Siemens Energy, Mülheim an der Ruhr, Germany for suggesting the topic of mutual benefit of Siemens Energy and the society at large. He was my constant support, provided me the necessary inputs and engaged in fruitful discussions for making this research meaningful for industrial use. I am grateful to him for providing me with esteemed platforms for presenting my research work. I am thankful to the management of Siemens and Siemens Energy with special mention of Mr. Mudit Jain, Mr. Rajiv Singh Bais, Mr. Amar Singh, Mr. Ankur Verma and my peers for supporting me during my research work.

I am indebted for life to my ‘Sir’, Mr. Rajib Banerjee, who is my friend, philosopher and guide. He sowed the seeds of my interest in Mathematics, Science and most importantly Ethics

when I was a child. I still live by and reap what I have learnt from him. I am thankful to all the passionate teachers who have taught me in my life.

I thank my parents-in-law for raising their lovely daughter. Their encouragement, moral support and patience during the tenure of my research is praiseworthy. I would like to thank my brothers-in-law, Mr. Sujoy Saha, Mr. Devraj Dutt and their respective families for their support and well wishes for my research. My brother, Mr. Manish Dhali and his family has been a life support for me and my family in Delhi.

My wife, the pillar of the family, has been the source of vital inspiration and critical strength in the entire duration of the Ph.D. program. She has silently made countless sacrifices, big and small, to see this day. She has been a lifeline and helped me sail through the high and low tides. Without her daily inspiration, support and sacrifices, this work would not have been possible. The two innocent diamonds of my life, our son Shrihan and daughter Prisha, have unknowingly sacrificed the time which I owed to them and let me focus on my research work. Their smiles were always refreshing and energizing especially during the stressful phases.

The power of the silent blessings of my maternal and paternal grandparents from their heavenly abode is ever priceless. No words are sufficient to express gratitude to the sacrifices of my parents, who held my hands, taught me little and big things in life, nurtured me, loved me, encouraged me to be what I wanted to be, made me capable, blessed me and set me free one day to pursue my dreams. My sister, my childhood friend, has been a source of constant love, inspiration and guidance in life. I would like to thank each of them for bringing me up and shaping my character of honesty and integrity.

The blessings of the Almighty were a constant source of power to propel myself in the right trajectory and overcome the difficulties during times of turbulence.

Suvadeep Sen

Abstract

The historic and everlasting aim to increase the steam temperature and pressure in steam turbines, to maximize efficiency, results in increased rate of creep/fatigue induced damage. To reduce the carbon footprint and meet the energy requirements at the same time, combination of conventional and renewable energy sources, with the predominant contribution of solar energy, is necessary. This requires flexible operation of conventional fossil fuel based power plants to compensate for the natural variation of the renewable energy availability leading to further enhancement in fatigue/creep damage. The non-linear interaction of creep and fatigue accelerates the total damage progression. The widely used uncoupled analysis using linear damage accumulation rules, wherein the damage due to creep and fatigue are considered separately and added in a linear manner, leads to conservative estimates of damage. To enable the flexible operation, advanced life assessment techniques are necessary for steam turbine rotors. Over the past two decades, studies using damage coupled unified constitutive models for the creep-fatigue interaction analysis of steam turbine components under complex loading cycles have attracted the attention of researchers/designers.

The constitutive model chosen in the present study involves the modified Chaboche non-linear kinematic and Chaboche and Rousselier isotropic hardening models with evolving damage, Norton type visco-plastic flow model, Lemaitre's damage potential function and modified form of Kachanov-Rabotnov's creep damage law. However, high computational time involved in the unified constitutive model based finite element analysis using iterative techniques hinder their widespread applications. In the present work, a non-iterative Asymptotic Numerical Method (ANM) for cyclic elasto-/visco- plasticity problems including damage evolution capable of handling multiple complex loading cycles is proposed and its application for steam turbine rotors is explored. To apply the polynomial expansion based ANM, several new regularizations of the governing equations/variables are proposed for multi-cycle elasto-/visco- plasticity. The methodology is implemented in ABAQUS through user material subroutine (UMAT). The accuracy and computational efficiency of the proposed method are tested considering results available in literature and comparison with the iterative Newton Raphson (NR) predictions for elasto- and visco- plastic analyses of steam turbine rotors under thermomechanical loading.

However, the expected computational time for the analysis of the complete rotor life is of the order of years even using the proposed ANM. To reduce the computational time to acceptable limits, a novel Representative Input Cycle (RIC) concept based local ANM is proposed for the complete life cycle analysis of steam turbine rotors. A long-term progressive damage analysis using the proposed RIC based ANM of a rotor under normal and normal followed by flexible operating regime is carried out. Reprofilng of the TSRG of the in-service rotor is investigated for enhancing the rotor life by a priori damage analysis.

Based on the analyses, it is found that ANM predictions are in excellent agreement with the Newton Raphson method. RIC based local ANM predicts normalized number of cycles to reach damage equal to unity in close agreement with the full finite element predictions and significantly smaller computational time requirement. The progressive damage analysis of a steam turbine rotor at a critical location employing the proposed RIC based ANM leads to computational time of ~5 days (including time for FE analysis for the initial loading period to construct the RIC based strain history) for 30 years of loading duration. Finally, the re-profiling of the thermal stress relief groove (TSRG) of the rotor decided through the RIC based progressive damage analysis predicted a life enhancement of ~31 years.

सारांश

दक्षता को अधिकतम करने के लिए, भाप टरबाइनों में भाप के तापमान और दबाव को बढ़ाने के ऐतिहासिक और चिरस्थायी उद्देश्य के परिणामस्वरूप सतत विकृति/विश्रांति प्रेरित भाप टरबाइनों में क्षति की दर में वृद्धि अवश्यभावी है। कार्बन पदचिह्नों को कम करने के साथ साथ ऊर्जा आवश्यकताओं को पूरा करने के लिए, पारंपरिक और सौर ऊर्जा के प्रमुख योगदान के साथ नवीकरणीय ऊर्जा स्रोतों का संयोजन आवश्यक है। इसके लिए नवीकरणीय ऊर्जा उपलब्धता की प्राकृतिक स्वभाविक भिन्नता के प्रभाव को कम करने के लिए पारंपरिक जीवाश्म ईंधन आधारित बिजली उत्पादन संयंत्रों के परिवर्तनीय संचालन की आवश्यकता होती है, जिससे विश्रांति/सतत विकृति क्षति में और भी वृद्धि होती है। सतत विकृति और विश्रांति की अरेखीय पारस्परिक प्रभाव कुल क्षति की प्रगति को तेज करता है। रैखिक क्षति संचय नियमों का उपयोग करते हुए व्यापक रूप से उपयोग किया जाने वाला अनयुग्मित विश्लेषण, जिसमें सतत विकृति और विश्रांति के कारण होने वाली क्षतियों को अलग-अलग आकलित किया जाता है और सीधे जोड़ा जाता है, कुल क्षति का आकलन अधिक करता है। परिवर्तनीय संचालन के लिए, भाप टरबाइन घूर्णक की उन्नत जीवन मूल्यांकन तकनीक आवश्यक है। पिछले दो दशकों में, जटिल बलचक्रों के तहत भाप टरबाइन घटकों के सतत विकृति-विश्रांति के पारस्परिक प्रभाव के विश्लेषण के लिए क्षति युग्मित एकीकृत प्रतिबल-विकृति संबंधों का उपयोग करने वाले अध्ययनों ने शोधकर्ताओं/अभिकल्पकों का ध्यान आकर्षित किया है।

प्रस्तुत अध्ययन के लिए चुने गए प्रतिबल-विकृति संबंधों में क्षति युग्मित संशोधित चाबोश अरेखीय किनेमैटिक और चाबोश और रूसेलियर आइसोट्रोपिक हार्डनिंग प्रतिरूप, नॉर्टन प्रकार का विस्कोप्लास्टिक प्रवाह प्रतिरूप, लेमैत्रे का क्षति विभव फलन और काचानोव-रबोटनोव का संशोधित सतत विकृति क्षति प्रतिरूप शामिल हैं। हालाँकि, पुनरावृत्त तकनीकों का उपयोग करके एकीकृत प्रतिबल-विकृति संबंधों पर आधारित फाइनाइट इलीमेन्ट विश्लेषण में अत्याधिक कम्प्यूटेशनल समय उनके व्यापक अनुप्रयोगों में बाधा है। प्रस्तुत शोध में, कई जटिल बलचक्रों के प्रभावों का आकलन करने में सक्षम क्षति विकास युग्मित चक्रीय इलास्टो-/विस्को-प्लास्टिसिटी के लिए एक गैर-पुनरावृत्त एसिम्प्टोटिक गणना विधि (एएनएम) प्रस्तावित की गयी है और इसका उपयोग भाप टरबाइन घूर्णक के विश्लेषण के लिए किया गया है। बहुपद विस्तार आधारित एएनएम को लागू करने के लिए, बहु-चक्रीय इलास्टो-/विस्को-प्लास्टिसिटी के गवर्निंग समीकरणों/चरों के कई नए नियमितीकरण प्रस्तावित किये गये हैं। इस विश्लेषण तरीके को अबाकस (ABAQUS) में युजर

मेटेरियल सबरूटीन (यूमैट) के माध्यम से कार्यान्वित किया गया है। प्रस्तावित विधि की सटीकता और कम्प्यूटेशनल दक्षता का परीक्षण साहित्य में उपलब्ध परिणामों से तुलना और थर्मोमैकेनिकल बलों के तहत भाप टरबाइन घूर्णक के इलास्टो- और विस्को-प्लास्टिक विश्लेषण के लिए पुनरावृत्त न्यूटन रैफसन (एनआर) आकलन के साथ तुलना करके किया गया है।

हालाँकि, प्रस्तावित एएनएम का उपयोग करते हुए भी संपूर्ण घूर्णक जीवन के विश्लेषण के लिए अपेक्षित कम्प्यूटेशनल समय वर्षों के क्रम का है। कम्प्यूटेशनल समय को स्वीकार्य सीमा तक कम करने के लिए, भाप टरबाइन घूर्णक के संपूर्ण जीवन चक्र विश्लेषण के लिए एक अभिनव प्रतिनिधि इनपुट चक्र (आरआईसी) अवधारणा आधारित लोकल (गॉस बिन्दु पर) एएनएम प्रस्तावित की गयी है। साधारण और उसके बाद परिवर्तनीय संचालन व्यवस्था के तहत घूर्णक के प्रस्तावित आरआईसी आधारित एएनएम का उपयोग करके दीर्घकालिक क्रमागत क्षति विश्लेषण किया गया है। प्राथमिक क्षति विश्लेषण द्वारा घूर्णक जीवन को बढ़ाने के लिए इन-सर्विस घूर्णक के टीएसआरजी की रीप्रोफाइलिंग की जांच की गयी है।

विश्लेषणों के आधार पर यह पाया गया कि एएनएम के आकलन न्यूटन रैफसन पद्धति के आकलन के साथ हबहू मिलते हैं। आरआईसी आधारित लोकल (गॉस बिन्दु पर) एएनएम पूर्ण फाइनाइट इलीमेन्ट आकलन के लगभग समान है और काफी कम कम्प्यूटेशनल समय लगता है। प्रस्तावित आरआईसी आधारित एएनएम का उपयोग करके भाप टरबाइन घूर्णक के एक नाजुक स्थान पर 30 वर्षों के क्रमागत क्षति विश्लेषण के लिए लगभग 5 दिनों का कम्प्यूटेशनल समय (आरआईसी आधारित विकृति इतिहास के निर्माण के लिए प्रारंभिक लोडिंग अवधि में फाइनाइट इलीमेन्ट आकलन के लिए समय सहित) लगता है। अंत में, आरआईसी आधारित क्रमागत क्षति विश्लेषण के माध्यम से एक घूर्णक के थर्मल स्ट्रेस रिलीफ ग्रूव (टीएसआरजी) की री-प्रोफाइलिंग से लगभग 31 साल की उपयोगिता में वृद्धि की भविष्यवाणी की गयी है।

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Nomenclature and Abbreviations

English Notations

a	Path parameter in ANM
A_c	Constant for creep damage
b	Constant representing speed of saturation in isotropic hardening
\mathbf{B}	Strain displacement matrix
\mathbf{C}^{el}	Elastic constitutive matrix of undamaged material
$\bar{\mathbf{C}}^{el}$	Modified Elastic constitutive matrix of undamaged material used in the regularized form of elastoplastic strain rate
C_l	Constants of kinematic hardening
\mathbf{C}^{tan}	Tangent stiffness matrix of the continuum with damage
\mathbf{d}^e	Vector of elemental degrees of freedom
D, D_c, D_f	Total Damage, Creep damage, Fatigue damage
$e_{\sigma_{Mises}}^{nRMS}$	Normalized root mean square error metric of von Mises stress
e_p^{nRMS}	Normalized root mean square error metric of Equivalent inelastic strain
e_D^{nRMS}	Normalized root mean square error metric of Damage
E	Young's modulus
f	Regularized form of the normalized flow potential (\bar{f})
\bar{f}	Normalized flow potential function
f_{nv}	Overstress
\bar{f}_{nv}	Regularized Overstress
f_v	Function to identify the loading regime
f_Y	Function representing the flow surface
F	Function of f, \bar{f} ensuring flow rule in the regularized expression of inelastic strain rate
\bar{F}_ω	Centrifugal force of each blade exerted on the rotor
$\bar{\mathbf{F}}_b$	Body force
G	Regularized form of β

$\bar{\mathbf{G}} (= G\mathbf{n})$	Variable used in the regularized expression of viscoplastic strain rate
$\mathbf{G}_{ep}^{\mathbf{n}} (= \mathbf{G}_{ep} : \mathbf{n})$	Variable used in the regularized expression of elastoplastic strain rate
h	Heat transfer coefficient
H	Unloading function
\mathbf{I}	Fourth order Identity tensor
\mathbf{I}'	Second order Identity tensor
J_2	Second invariant of deviatoric part of $\tilde{\boldsymbol{\sigma}} - \mathbf{X}$
k_c	Creep damage exponent (damage term)
K_v	Coefficient of the Norton's viscoplastic flow rule
\mathbf{K}^e	Elemental stiffness matrix
L	Normalized loading function
\bar{L}	Regularized form of the normalized loading function (L)
\mathbf{m}	Normal of a boundary element
M	Variable ensuring correct viscous flow/dissipation
n_v	Viscosity exponent
\mathbf{n}	Regularized form of \mathbf{n}_0
\mathbf{n}_0	Unit normal to the inelastic flow potential surface
N	Number of cycles
N^*	Normalized number of cycles
\dot{p}	Regularized form of equivalent inelastic strain rate (\dot{p}_v)
\dot{p}_v	Equivalent inelastic strain rate
q	Regularized form of the second invariant of deviatoric part of $\tilde{\boldsymbol{\sigma}} - \mathbf{X}$
Q	Asymptotic value of isotropic hardening
r_c	Creep damage exponent (stress term)
R	Isotropic hardening with evolving damage
R^*	Isotropic hardening
\bar{R}	Radius of rotor
s	Fatigue damage exponent
S	Fatigue damage constant
\check{S}^e	Boundary surface of a finite element

S_{y_0}	Initial yield stress
t	Time
t_0	Time at the end of previous converged step
Δt	Time increment
T	Total loading time
\mathbf{T}_m	Surface traction acting on the boundary of a finite element
u	Displacement
U	Set of all variables used in ANM
U_0	U at the end of previous ANM step
\hat{U}_k	k^{th} order coefficient of U
V	$= \sigma_v^{n_v}$
\check{V}^e	Volume of a finite element
\mathbf{X}	Back stress tensor with evolving damage
\mathbf{X}^*	Back stress tensor
\mathbf{X}'	Deviatoric part of back stress tensor
$\bar{\mathbf{X}}$	Fourth order tensor composed \mathbf{X} and \mathbf{n}
Y_e	Elastic energy release rate
Y	Regularized form of elastic energy release rate
\bar{Y}	$= Y^s$

Greek Notations

β	Regularized form of F
γ_l	Constants for non-linear kinematic hardening
Γ	Total number of time steps
Γ^*	Normalized computation time
Γ_c	Computation time
ϵ	Total strain tensor
ϵ^e	Elastic strain tensor
ϵ^p	Inelastic strain tensor

ζ	Force vector in finite element formulation
η	Regularization parameter
θ	Temperature
$\Delta\theta$	Temperature difference
θ_{rated}^s	Rated steam temperature
θ_{av}	Sectional average temperature
λ	Plasticity multiplier
σ	Nominal stress tensor
$\tilde{\sigma}$	Effective stress tensor
$\tilde{\sigma}'$	Deviatoric part of effective stress tensor
$\tilde{\sigma}_H$	Hydrostatic part of effective stress tensor
σ_v	Regularized form of regularized overstress (\bar{f}_{nv})
σ_e	$= R + S_{y_0} + \sigma_v$
Ψ	Displacement interpolation matrix
ω	Angular speed

Abbreviations

ANM	Asymptotic Numerical Method
ASME	The American Society of Mechanical Engineers
CDM	Continuum Damage Mechanics
CEA	Central Electricity Authority of India
CPU	Central Processing Unit
CS/WS/HS/MLC	Cold Start/Warm Start/Hot Start/Major Load Change
FE/FEM	Finite Element / Finite Element Method
FOR	Flexible Operating Regime
HIS	Complete Strain History
HP	High Pressure
HTC	Heat Transfer Coefficient
IP	Intermediate Pressure
LATIN	Large Time Increment Method

LCF	Low Cycle Fatigue
MTL	Minimum Thermal Load
NOR	Normal Operating Regime
NR	Newton Raphson
PGD	Proper Generalized Decomposition
RES	Renewable Energy Sources
RIC	Representative Input Cycle
RMS/nRMS	Root Mean Square/ Normalized Root Mean Square
RVE	Representative Volume Element
SARA	Schadens-Akkumulation-Rechner-Anwendung (Software for Lifetime Estimation with Accumulative Model)
TSRG	Thermal Stress Relief Groove
UCM	Unified Constitutive model
UMAT	User Material Subroutine