

MULTIRATE SENSOR ARRAY PROCESSING

by

RAVINDRA DHULI

DEPARTMENT OF ELECTRICAL ENGINEERING

Submitted

in fulfilment of the requirements of the degree of Doctor of Philosophy

to the



INDIAN INSTITUTE OF TECHNOLOGY DELHI

MARCH 2010

Certificate

This is to certify that the thesis titled “**Multirate Sensor Array Processing**” being submitted by Ravindra Dhuli to the Department of Electrical Engineering, Indian Institute of Technology, Delhi, for the award of the degree of Doctor of Philosophy, is a record of bona-fide research work carried out by him under my guidance and supervision. In my opinion, the thesis has reached the standards fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted to any other university or institute for the award of any degree or diploma.

Dr. Brijesh Lall
Assistant Professor
Department of Electrical Engg.,
Indian Institute of Technology, Delhi
New Delhi - 110 016

To
Our Beloved Joshi Sir,
A Great Saint I met in my life.

Acknowledgments

First, I would like to sincerely thank my supervisor, Dr. Brejesh Lall for making my research a beautiful journey. I can never forget the way he encouraged me with his wonderful smiles. He helped me to sail through the infinite ocean that appeared before me during my initial stages of research. His magic words are “Ph.D is all about *Learning how to learn!!*”.

I would also like to thank my SRC members, Prof. R. K. Patney, Prof. S. D. Joshi and Dr. Monika Aggarwal for their valuable suggestions in my research presentations. Prof. Shiv Dutt Joshi motivated me into the field of signal processing through the first lecture that I attended in IIT Delhi. He changed my fundamental way of looking at signals and life altogether. He says “*Listen to your **Heart***”.

I am grateful to the helpful correspondence with Prof. Bruce A. Francis and Dr. Omid. S. Jahromi. These communications were of great help during my crucial stage of research.

Friends at IIT Delhi broadened my thinking in all aspects. I thank each and every one of them for their contribution.

Love makes life beautiful. The immense love and infinite sacrifices from the part of my family members has brought alive my dream of attaining a Ph.D. I may never be able to repay them in any form.

New Delhi

Ravindra Dhuli

Abstract

A phenomenon is observed by a multirate sensor array, constituting sensors each operating at its own sampling rate. From the low resolution observations of the sensor array, one has to estimate the signal of interest at a higher resolution. To achieve this we develop a digital system model for the sensor array. Studying various issues in modeling the sensor array as a nonuniform filter bank (NUFB), analysis results are presented. We then develop equivalent uniform filter bank (UFB) representation of this NUFB model. A generalisation of pseudocirculants, giving the necessary and sufficient conditions for the alias reduction are derived. Thus, showing that these results are useful in simplifying the LPTV systems. We further present the extension of the generalised pseudocirculants for the analysis of general linear shift invariant (LSIV) systems and cyclostationary signals. In course of our theoretical developments, the generalised polyphase identity is obtained. We show how these results are useful in understanding the behaviour of the re-sampled 2-D periodic signals.

Statistical analysis of multirate systems is considered later. We show that TFR combined with blocking is a simple yet efficient tool for such analysis. Using the standard polyphase decompositions we also show how this analysis fits naturally in a framework of multirate systems.

Finally, the problem of data-fusion of the sensor array is addressed for two specific scenarios. In the first scenario the analysis stage of NUFB is used as the model. In the second scenario, to handle the sensor array with fractional delays, we use a hybrid NUFB model. We design, the system using the minimax optimisation. Using the general LSIV systems, we reduce the NUFB design problem to that of a UFB, thereby removing the structural dependencies in the synthesis stage. As a part of this analysis, the design relations developed for efficient realisations of the synthesis systems are also presented.

The contribution of this thesis can be viewed as the development of techniques that aid in the study and analysis of multirate sensor array processing. The specific contributions of this thesis can be divided into the broad areas of:

- (i) multirate signal processing theory
- (ii) data-fusion algorithms.

- Our work first presents an analysis of an existing model for the multirate sen-

sensor array. We then present the generalisation of the pseudocirculants and the polyphase identity. Showing that time frequency representation (TFR) is a natural choice for statistical analysis of the multirate systems is also discussed.

- For the design of the data-fusion system, we use the \mathcal{H}_∞ -optimisation where the worst case error norm is minimised. This design technique is extended to handle the more general scenario of fusing the data from the sensor array with fractional delays.

Contents

Acknowledgments	v
Abstract	vii
Contents	ix
List of Figures	xiii
List of Abbreviations	xv
1 Introduction	1
1.1 Literature survey	4
1.2 Approach and Contributions	8
1.3 Thesis Organisation	10
2 Preliminaries	13
2.1 Introduction	13
2.2 Review of multirate basics	14
2.2.1 Downsampling and Upsampling	14
2.2.2 Noble Identities	15
2.2.3 Polyphase Decompositions	16
2.3 Converting NUFB to an equivalent UFB	17
2.4 Multirate sensor array	18
2.4.1 Intertwining the sampling operation with the system operations	20
2.5 Concluding Remarks	22
3 Alias reduction: Generalised Pseudocirculant Conditions	23
3.1 Introduction	24

3.1.1	Notations	26
3.2	Insight into LPTV system	26
3.2.1	Switching representations	26
3.2.2	Alias terms	28
3.3	System Simplification	30
3.4	Switching Representations for Filter bank	32
3.5	Generalised Pseudocirculant Conditions	35
3.5.1	Conditions on the matrix $\mathbf{A}(z)$ for alias reduction	36
3.5.2	Example	37
3.6	Conclusion	39
4	Further Extensions to Generalised Pseudocirculants	41
4.1	Introduction	41
4.2	General LSIV systems	42
4.2.1	Blocking operation	43
4.2.2	Blocking Implementation for High speed Digital filtering	44
4.3	Relation between the Blocking operators	46
4.3.1	Review of MIMO representation	46
4.3.2	Relating the blocking operators	47
4.4	Generalised pseudocirculant conditions	48
4.5	Conclusions	52
5	Re-sampling 2-D periodic signals: Generalising the Polyphase Identity	53
5.1	Introduction	54
5.2	Re-sampling Cyclostationary Signals	56
5.2.1	Downsampling Cyclostationary Signals	57
5.2.2	Block Downsampling Cyclostationary Signals	58
5.2.3	Upsampling Cyclostationary Signals	59
5.3	Generalised Polyphase Identity 1	60
5.3.1	Downsampling 1-D Periodic Signal	61
5.3.2	Using blocksamplers	65
5.4	Generalised Polyphase Identity 2	66
5.5	Conclusion	68

6	Cyclostationary signals in multirate linear systems	69
6.1	Introduction	70
6.1.1	Notations	71
6.2	New insights into the blocking operation	71
6.3	Generalised pseudocirculants: Cyclostationary Signals	75
6.4	TFR and Power Spectrum Matrix	78
6.5	Examples	82
6.5.1	Decimation	82
6.5.2	Interpolation	83
6.5.3	Dual-rate system	84
6.5.4	An NUFB	85
6.6	Conclusions	86
7	Multirate Sensor Array Data-fusion	87
7.1	Introduction	88
7.1.1	Notation	90
7.2	General LSIV synthesis systems	90
7.2.1	Design Criterion	92
7.3	Design of efficient realisations of the general synthesis systems	95
7.3.1	The First Realisation	96
7.3.2	The Second Realisation	98
7.4	Designing IIR Synthesis Systems	100
7.4.1	Design Procedure	101
7.4.2	Simulation Results	103
7.5	Design of FIR synthesis systems	106
7.5.1	Modeling as a Linear Matrix Inequalities	107
7.5.2	Design procedure	108
7.5.3	Simulation Results	109
7.6	Conclusions	110
8	Data-fusion: Multirate sensor array with fractional Delays	117
8.1	Introduction	117
8.1.1	Notation	119
8.2	Converting Hybrid NUFB into digital NUFB	119
8.3	Design of the synthesis stage	125

8.3.1	Design procedure	125
8.3.2	Simulation Results	127
8.4	Conclusions	131
9	Concluding remarks	137
9.1	Scope for future research	138
A	State space representations in	
	Multirate Signal Processing	141
A.1	State Space representation	142
A.1.1	Representation of SISO LTI system	143
A.1.2	Polyphase Decompositions	144
A.1.3	Representation of $\mathbf{H}(z^m)$	146
	Bibliography	148
	Publications	155
	Bio-Data of the Author	157