

# IMPACT MODELING OF POLYMER COMPOSITES USING MACROSCALE AND MULTISCALE TECHNIQUES

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INDIAN INSTITUTE OF TECHNOLOGY DELHI

October, 2017

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**IMPACT MODELING OF POLYMER  
COMPOSITES USING MACROSCALE AND  
MULTISCALE TECHNIQUES**

by

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Submitted

In fulfillment of the requirements of the degree of Doctor of Philosophy

to the



INDIAN INSTITUTE OF TECHNOLOGY DELHI

October, 2017

*Dedicated to my beloved parents and family*

# Certificate

This is to certify that the thesis entitled “**Impact Modeling of Polymer Composites Using Macroscale and Multiscale Techniques**” being submitted by **Mr. Harpreet Singh** is the report of bonafide research work carried by him under my supervision. This thesis has been prepared in conformity with the rules and regulations of INDIAN INSTITUTE OF TECHNOLOGY DELHI. I further certify that the thesis has attained a standard required for the award of doctor of philosophy degree of the Institute. The research reported and the results presented in the thesis have not been submitted, in part or full to any other institute or university for the award of any other degree or diploma.

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Place: New Delhi

# Acknowledgements

Before expressing my gratitude to those without whom it would not be possible for me to be as I am, I would like to share my feelings for these wonderful and unforgettable past four years of my life. I would like to say that Ph.D. is not an achievement it is a journey. After going through this I felt lot of changes in me and came out as a committed, sincere and a deep thinker.

For that, I would like to express my deepest gratitude to my research guru, my advisor **Prof. Puneet Mahajan** for his guidance, support and encouragement without which the work would have not been realized. The discussions those happened on each Saturday either at his office or over coffee with him helped me to strengthen my computational mechanics knowledge and accomplish my goals. I also admire him for being very patient, understanding and providing me the guidance on all professional or personal fronts.

I am really thankful to my wife, **Mandeep Kaur** for supporting me in everything. Her love, care and wisdom make me feel enriched and gives me a sense of completeness in my life. I also acknowledge my daughters **Guneet Kaur (Nonu)** and **Prabhnoor Kaur (Nooru)** for their love and for being part of my life although my appologies for not spending enough time with them. These three always remind me what is important in my life.

Next, I thank all faulty members of Department of Applied Mechanics who played an important role in my research experience at IIT Delhi. I am also thankful to all my colleagues of Impact Mechanics lab specially **Dr. Sangeeta Khare** for her valueable support.

I am really blessed to have **Mohit Gupta** and **Anurag Kumar Singh** as friends during this tenure. Their great support and technical discussions helped me to enrich my knowledge regarding my research topic. No amount of words would suffice to thank them for their favor and cooperation. I am especially grateful to **Kiran Kumar Namala** and **Mohit Gupta** for conducting all the exeperiments related to my research.

## *Acknowledgements*

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I would also like to thank all my colleagues and friends for their support and cooperation during this period.

I am also grateful to my mother **Smt. Harbir Kaur** and my father **S. Kulwant Singh** for their support and blessings. I also thank to my brother **Damanpreet Singh** and his family for their motivation and support.

**Above all, the Almighty.**

Harpreet Singh

# Abstract

Composite materials, which are being used in aerospace industry, are susceptible to impact during their service life. In the present work, numerical and analytical techniques for modeling the impact response are discussed.

Based on the scale these techniques can be categorized as macroscale and multiscale. In macroscale model, the heterogeneous material is replaced by an effective homogenized material at lamina level. In the present research work both the macroscale and microscale modeling techniques are investigated. Low velocity impact simulations for E-glass/epoxy composite are performed using continuum damage mechanics (CDM) based macroscale material model. These studies consider the elastic deformations and intralaminar/interlaminar damage evolves exponentially or linearly with strain in these models. The choice of material softening parameter, ' $m$ ' for a particular mesh size is discussed and determined from the FE simulation of a single element model. The damage observed through the light projected area on the laminate, contact forces and displacement plots with respect to time are studied and compared with finite element analysis results to demonstrate the effectiveness of the model.

Further, an elasto plastic damage model for three dimensional fiber reinforced polymer (FRP) composite is proposed to simulate the progressive damage and damaged induced inelastic deformation in case of low velocity impact. Exponential softening model is implemented for damage growth prediction and a three dimensional plastic potential is used for plastic surface growth. A user subroutine, VUMAT is written and implemented in ABAQUS/Explicit. Low velocity impact simulations are performed with graphite/epoxy laminate. Delamination behavior is simulated using surface based cohesive interaction between the adjacent laminae. The results show that the predicted behavior matches well with experimental observations. The model is successfully able to capture the central deflection

of the laminate plate and permanent indentation on the surface.

An improved analytical model to predict the response of composite laminate under low velocity large mass impact is proposed. A spring–mass system is used to represent the contact, bending, shear and membrane stiffnesses of the laminate system. The stiffness of the springs is evaluated by dividing the laminate into a central damaged region with degraded stiffness surrounded by an undamaged region. The stiffness of the springs changes as the damage region grows with time. The predictions of the force between impactor and laminate and size of the damaged region from the proposed model are compared with the FE simulation results and available experimental results in the literature for CFRP laminates. The comparison demonstrates a good prediction capability of the proposed formulation.

The structural analysis of a composite material undergoing degradation in the properties can also be done using the multiscale approach. A multiscale procedure based on asymptotic expansion homogenization (AEH) technique is formulated to study the inelastic deformations and damage of FRP composite materials. The complex implementation and high computational cost of AEH can be alleviated by the reduced order techniques. In the present work, the reduced order AEH formulation is extended to capture the plasticity and damage in the matrix. The initial studies are performed for representative volume element (RVE) in absence of debonding which show the expected behavior. This new formulation is applied to study the low velocity impact of composites. The results (force-time curves and extent of damage) are validated with experiments. The model is successfully able to capture the deflection at various points on the laminate and permanent indentation on the surface. The effect of debonding of fiber-matrix interface was included in past by using eigen deformation based formulation. Here, an alternate method is proposed to include the debonding effects by modeling it as an additional interphase in RVE. Finally the low velocity impact simulation of a composite plate is carried out for the validation of proposed two scale formulation. The importance of including the debonding effects is also demonstrated in terms of significant difference in the results calculated by ignoring these effects.

# सार

कम्पोजिट सामग्री, जिसका उपयोग एयरोस्पेस उद्योग में किया जा रहा है, उनकी सेवा जीवन के दौरान प्रभावित होने की संभावना है। वर्तमान कार्य में, प्रभाव प्रतिक्रिया के मॉडलिंग के लिए संख्यात्मक और विश्लेषणात्मक तकनीकों पर चर्चा की है।

पैमाने के आधार पर इन तकनीकों को मैक्रोस्केल और मल्टीस्केल के रूप में वर्गीकृत किया जा सकता है। मैक्रोस्केल मॉडल में, विषम पदार्थ को लेमिना स्तर पर एक प्रभावी होमोजीनाइज्ड सामग्री द्वारा प्रतिस्थापित किया जाता है। वर्तमान शोध कार्य में मैक्रोस्केल और माइक्रोस्कोल मॉडलिंग तकनीक दोनों की जांच की है। ई-ग्लास / इपॉक्सी कम्पोजिट के लिए कम वेग प्रभाव सिमुलेशन, सातत्य क्षति यांत्रिकी (सीडीएम) आधारित मैक्रोस्केल सामग्री मॉडल का उपयोग करके किया है। इन अध्ययनों में लोचदार विरूपताओं पर विचार किया है और इन मॉडलों में इंटर-लामिना / अंतर-लामिना क्षति तनाव के साथ तेजी से या रैखिक विकसित होते हैं। एक विशेष आकार के जाल के लिए सामग्री नरम पैरामीटर, 'm' के चुनाव पर चर्चा की है और एक एकल तत्व मॉडल के FE सिमुलेशन से निर्धारित किया है। समय के बदलते संपर्क बलों, विस्थापन के भूखंडों और प्रकाश प्रक्षेपित क्षेत्र के माध्यम से निकाले हुए नुकसान का अध्ययन किया है और मॉडल के प्रभावशीलता का प्रदर्शन करने के लिए FE विश्लेषण परिणामों के साथ तुलना की है।

इसके अलावा, तीन आयामी फाइबर प्रबलित पॉलिमर (एफआरपी) संमिश्र के लिए निम्न वेग प्रभाव के मामले में हुए प्रगतिशील क्षति और क्षतिग्रस्त निर्बाध विरूपण का अनुकरण करने के लिए एक इस्तास्टो प्लास्टिक क्षति मॉडल प्रस्तावित किया है। घातीय नरम मॉडल क्षति वृद्धि की भविष्यवाणी के लिए लागू किया है और प्लास्टिक की सतह के विकास के लिए एक तीन आयामी प्लास्टिक संभाव्य का उपयोग किया है। एक यूजर सब-रूटिन, VUMAT को ABAQUS/Explicit में लिखा और कार्यान्वित किया गया है। कम वेग प्रभाव सिमुलेशन का उपयोग ग्रेफाइट / इपोकिक संमिश्र किया है। सटे-लैमिना के बीच की सतह पर दुर्गम व्यवहार का विरूपण सतह आधारित cohesive interaction का उपयोग करके किया है। परिणाम बताते हैं कि अनुमानित व्यवहार प्रयोगात्मक अवलोकन के साथ अच्छी तरह से मेल खाता है। मॉडल सफलतापूर्वक प्लेट की सतह पर स्थायी खरोज और केंद्रीय नीचे को झुकाव को कैप्चर करने में सक्षम है।

कम वेग बड़े वजन के तहत लैमिनेट की प्रतिक्रिया का अनुमान लगाने के लिए एक बेहतर विश्लेषणात्मक मॉडल का प्रस्ताव किया है। स्प्रिंग-वजन प्रणाली का उपयोग लैमिनेट के संपर्क, झुकाव, कतरनी और झिल्ली कठोरता का प्रतिनिधित्व करने के लिए किया है। स्प्रिंग्स की कठोरता का मूल्यांकन लैमिनेट को एक निर्बाध क्षेत्र से घिरे हुए एक केंद्रीय न्यून कठोर क्षतिग्रस्त क्षेत्र में विभाजित करके किया है। स्प्रिंग्स की कठोरता में परिवर्तन क्षति क्षेत्र के रूप में समय के साथ बढ़ता है। प्रस्तावक मॉडल से CFRP लैमिनेट के लिए निकाले हुए क्षतिग्रस्त क्षेत्र के आकार और लैमिनेट और इम्पैक्टर बल की FE सिमुलेशन के परिणाम और साहित्य में उपलब्ध प्रयोगात्मक परिणाम से तुलना की है। तुलना प्रस्तावित सूत्रीकरण की अच्छी भविष्यवाणी क्षमता दर्शाता है।

गुण में गिरावट के दौर से गुजरने वाली कम्पोजिट सामग्री का संरचनात्मक विश्लेषण मल्टीस्केल दृष्टिकोण का उपयोग करके भी किया जा सकता है। असिम्प्टोटिक विस्तार होमोजीनाइजेशन (AEH) तकनीक के आधार पर एक बहुक्रिया प्रक्रिया को FRP कम्पोजिट सामग्री के असंगत विकृति और नुकसान का अध्ययन करने के लिए तैयार किया गया है। जटिल क्रियान्वयन और AEH की उच्च कम्प्यूटेशनल लागत को कम ऑर्डर तकनीकों द्वारा कम किया जा सकता है। वर्तमान कार्य में, मैट्रिक्स में प्लास्टिसिटी और नुकसान को कैप्चर करने के लिए AEH फॉर्मूलेशन को विस्तृत किया है। शुरुआती

अध्ययन प्रतिनिधि मात्रा तत्व (REV) के लिए किया है जो देबॉडिंग की अनुपस्थिति में है जो अपेक्षित व्यवहार दिखाता है। कंपोजिट के कम वेग प्रभाव का अध्ययन करने के लिए यह नया निर्धारण लागू किया गया है। परिणाम (बल-समय वक्र और क्षति की सीमा) प्रयोगों के साथ मान्य हैं। मॉडल सफलतापूर्वक प्लेट की सतह पर स्थायी खरोज और विभिन्न बिंदुओं पर नीचे को झुकाव को कैचर करने में सक्षम है। Eigen विरूपण आधारित ढांचे का उपयोग करके पूर्व में फायर बिर-मैट्रिक्स इंटरफेस के देबॉडिंग का प्रभाव शामिल किया गया था। यहां, एक वैकल्पिक विधि द्वारा RVE में अतिरिक्त इंटरफेस के रूप में मॉडलिंग द्वारा देबॉडिंग प्रभाव शामिल करने का प्रस्ताव है। अंत में, प्रस्तावित दो पैमाने की सूत्रीकरण की पुष्टि के लिए एक कंपोजिट प्लेट के कम वेग प्रभाव सिमुलेशन किया है। देबॉडिंग के प्रभाव को शामिल करने के महत्व को इन प्रभाव की अनदेखी करके निकाले परिणामों में महत्वपूर्ण अंतर के संदर्भ में भी प्रदर्शित किया गया है।

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# List of Symbols

$\alpha, \beta$	Microscale partition number
$\mu_{ij}$	Eigen strain
$\xi$	A number representing ratio of macroscopic to microscopic dimension
$a_c$	Final crack size
$a_d$	Detectable crack size
$a_i$	Initial crack size
$a_{66}$	A parameter used for plastic potential function
$c$	Radius of damaged region of the plate
$d_{1c}$	damage variable for fiber compression
$d_{1t}$	damage variable for fiber tension
$d_{2c}$	damage variable for matrix/transverse compression
$d_{2t}$	damage variable for matrix/transverse tension
$f$	Plastic potential function
$h$	Plate thickness
$k_b$	Bending stiffness of the plate
$k_c$	Contact stiffness of the plate
$k_m$	Membrane stiffness of the plate

## List of Symbols

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$k_s$	Shear stiffness of the plate
$k_{bs}$	Combined bending and shear stiffness of the plate
$m$	Material parameter for defining the softening behavior
$m_I$	Mass of impactor
$m_P$	Mass of plate
$n$	Exponent for flow stress
$n_d$	Number of delaminations
$p$	Contact pressure
$p_{\perp\parallel}^{(+)}$	Parameter-1 used for Puck theory
$p_{\perp\parallel}^{(-)}$	Parameter-2 used for Puck theory
$p_{\perp\perp}^{(-)}$	Parameter-3 used for Puck theory
$p_i$	Traction on the surface
$r_c$	Contact radius
$t_e$	Thickness of the an element
$u_i$	Displacement
$w$	Deflection of the plate
$x_I$	Displacement of impactor
$x_P$	Displacement of plate
$k_c^D$	Contact stiffness of damaged portion of the plate
$k_c^{UD}$	Contact stiffness of undamaged portion of the plate
$A_D$	Damaged cross-section area
$A_o$	Undamaged cross-section area

$C_{CI}$	Constant for clamped-immovable membrane
$C_{SM}$	Constant for simply supported-movable membrane
$D$	Damage variable
$D^*$	Effective stiffness of orthotropic plate
$D^r$	Bending stiffness of the plate in the radial direction
$E_i$	Elastic Modulus in $i$ direction, $i=1, 2$ and $3$
$E_{ijkl}$	Elastic influence function
$F$	Force
$F_{11}^c$	Function for defining fiber direction compressive failure
$F_{11}^t$	Function for defining matrix direction compressive failure
$F_{11}^t$	Function for defining fiber direction tensile failure
$F_{22}^t$	Function for defining matrix direction tensile failure
$F_c$	Contact force between impactor and plate
$F_i$	Strength parameter used for Tsai-Wu theory
$F_p$	Plastic potential function
$F_{ij}$	Strength parameter used for Tsai-Wu theory
$F_{th}$	Threshold force for delamination
$F_{yield}$	Yield function
$G_{ij}$	Shear Modulus in $(i, j)$ plane, $ij = 12, 23$ and $13$
$K_{ij}$	Stiffness coefficient for delamination representing ratio of force in $i$ -direction to displacement in $j$ -direction
$L_c$	Characteristic length of an element

## List of Symbols

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$L_{ijkl}(\mathcal{Y})$	Constant for simply supported-movable membrane
$L_x$	Length of a brick element in $x$ -direction
$L_y$	Length of a brick element in $y$ -direction
$Q_z$	Stiffness parameter for contact stiffness
$R$	Radius of impactor
$S_{12}$	Shear strength in (1, 2) plane
$S_{13}$	Shear strength in (1, 3) plane
$S_{23}$	Shear strength in (2, 3) plane
$S_{ijkl}$	Damage influence function
$V_f$	Fiber volume fraction
$V_o$	Initial velocity of impactor
$W_{SE}$	Strain energy per unit volume
$X, Y, Z$	Strengths in $x, y$ and $z$ direction
$X_C$	Compressive strength in fiber direction
$X_T$	Tensile strength in fiber direction
$Y_C$	Compressive strength in transverse direction
$Y_T$	Tensile strength in transverse direction
$Y_c$	Compressive yield strength
$Y_t$	Tensile yield strength
$\bar{E}$	Effective modulus
$\bar{L}_{ijpq}$	Homogenized constitutive tensor
$\bar{M}_{ijpq}$	Homogenized tensor relating eigen strain to normal strain

$\mathbf{C}$	Stiffness tensor
$\mathbf{C}_o$	Stiffness tensor for undamaged material
$\mathbf{M}$	Damage effect tensor or Integrity tensor
$\mathcal{G}$	Fracture energy release rate
$\mathcal{G}_{III}$	Mode-III fracture energy release rate
$\mathcal{G}_{II}$	Mode-II fracture energy release rate
$\mathcal{G}_I$	Mode-I fracture energy release rate
$\mathcal{G}_S$	Fracture energy release rate due to shear tractions
$\mathcal{G}_c$	Critical fracture energy release rate
$\mathcal{G}_i^j$	Fracture energy release rate in phase $i$ due to $j$ type load
$\mathcal{X}$	Macroscopic domain
$\mathcal{Y}$	Microscopic domain
$C$	Stiffness coefficient
$\Delta$	Relative displacement between plies
$\bar{\sigma}$	Effective stress
$\boldsymbol{\sigma}$	Stress tensor
$\boldsymbol{\Omega}$	Damage tensor
$\boldsymbol{\varepsilon}$	Strain tensor
$\boldsymbol{\varepsilon}_e$	Elastic strain tensor
$\boldsymbol{\varepsilon}_p$	Plastic strain tensor
$\delta_{ij}$	Kronecker delta
$\dot{\lambda}$	Proportionality factor for plastic strains

## List of Symbols

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$\eta$	Power exponent for Benzeggagh–Kenane (BK) law
$\mathbf{x}$	A vector in macroscale
$\mathbf{y}$	A vector in microscale
$\mu$	Coefficient of friction
$\nu$	Poisson's ratio
$\sigma$	Stress
$\sigma^{eq}$	Equivalent stress
$\sigma_{flow}$	Flow stress
$\varepsilon$	Strain
$\varepsilon_e$	Elastic strain
$\varepsilon_f$	Failure strain
$\varepsilon_o$	Damage initiation strain
$\varepsilon_p$	Plastic strain
$(.)^D$	Quantity for damaged material
$(.)^{UD}$	Quantity for undamaged material
$(.)_n$	$n^{th}$ load step value

