

VIBRATION RESPONSE OF MULTILAYER STRUCTURES
TO RANDOM EXCITATION

by

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ABSTRACT

The present work deals with the analysis of multilayer beams and plates, subjected to random excitations. The multilayer configurations analysed are of the three-layer type with viscoelastic or 'elasto-dissipative' core and of the two-layer elastic-viscoelastic type. Two types of random excitations, namely, the white-noise random excitation and the turbulent boundary layer excitation have been considered. The theoretical work reported has been verified experimentally.

Equations of motion of a general 3-layer sandwich beam is first derived using the method of equilibrium of forces and the compatibility conditions. For a sandwich beam having a viscoelastic core, a 4-element model is chosen to represent the viscoelastic properties of the core material in shear. It is then incorporated in the equation of motion of the general 3-layer beam to obtain the governing equation for a beam having a viscoelastic core. The transfer function of the beam is determined from this equation. A series solution satisfying the simply supported end conditions is used and random vibration analysis applied to obtain an expression for the mean-square displacement response in terms of the transfer function.

For a sandwich beam having an 'elasto-dissipative' core, internal damping based on constant Q hypothesis is introduced for the core and face layers in the equation of motion of the general 3-layer beam to obtain the governing equation. The transfer function of the beam is determined from the governing

equation. The mean-square displacement response of the beam is now determined by using a series solution for the response, satisfying the simply supported end conditions, in conjunction with the random vibration analysis.

For a simply supported 2-layer elastic-viscoelastic beam arrangement, the equation of motion is obtained directly, by suitable substitutions, from the corresponding equations of the 3-layer sandwich beam having a viscoelastic core. The viscoelastic layer is idealized to a 4-element model to represent the properties of the viscoelastic material in direct strains. Mean-square displacement response of the beam is then obtained in the same manner as for the 3-layer beam.

The mean-square displacement response of the 3-layer plates having viscoelastic or 'elasto-dissipative' cores, and of the 2-layer elastic-viscoelastic plates is obtained on lines similar to those for the corresponding cases of the beams.

The integrals involved in the expressions of response are evaluated either by the method of residues or by a suitable numerical integration technique depending upon the nature of the integrand.

The effectiveness of various geometrical and physical parameters, in minimizing the response of the above-mentioned beams and plates is evaluated. Based on specific design criteria comparison of the response of a 3-layer sandwich beam having viscoelastic core is made with a reference homogeneous beam,

and the conditions, under which minimum response can be obtained, are established.

Experiments, conducted on a few samples of multilayer beams subjected to white-noise random and turbulent boundary layer excitations, are reported. The test results are compared with the corresponding theoretical results.

A general discussion on and conclusions about the present work, and the scope for future work are given at the end.

CERTIFICATE

This is to certify that the Thesis entitled "Vibration Response of Multilayer Structures to Random Excitation" being submitted by Mr. Omkar Nath Kaul to the Indian Institute of Technology, Delhi, for the award of the Degree of Doctor of Philosophy in Mechanical Engineering, is a record of bonafide research work carried out by him. He has worked under our guidance and supervision and has fulfilled the requirements for the submission of this Thesis, which has reached the requisite standard.

The results contained in this Thesis have not been submitted in part or in full, to any other University or Institute for the award of any degree or diploma.



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CONTENTS

		<u>Page</u>
ABSTRACT	(i)
CERTIFICATE	(iv)
ACKNOWLEDGEMENTS	(v)
CONTENTS	(vi)
TERMINOLOGY	(xi)
LIST OF FIGURES	(xiv)
LIST OF TABLES	(xix)
CHAPTER 1	INTRODUCTION, LITERATURE SURVEY AND OUTLINE OF THE PRESENT WORK	1
1.1	Introduction	1
1.2	Literature Survey	4
1.	Static Analysis of Multilayer Structures	4
2.	Dynamic Analysis of Multilayer Structures	4
1.	Harmonic Excitation	4
2.	Representation of Dynamic Properties of Viscoelastic Materials	11
3.	Shock Excitation	13
4.	Random Excitation	14
3.	Outline of the Present Work	27
CHAPTER 2	RESPONSE OF THREE-LAYER SANDWICH BEAMS TO RANDOM EXCITATION	31
2.1	Introduction	31
2.2	Assumptions	31

2.3	Equations of Motion of a Three-layer Sandwich Beam	32
2.4	Response of a simply-supported Sandwich Beam, having Viscoelastic Core, to Random Excitation.	42
1.	Four-element Viscoelastic Model	43
2.	Equation of Motion	43
3.	Transfer Function of the System	44
4.	White-noise Random Excitation of Distributed Load Type.	45
5.	White-noise Random Excitation of Point Load Type	51
6.	Turbulent Boundary Layer Excitation	56
2.5	Results and Discussion-Beams with Viscoelastic Core	67
1.	Introduction	67
2.	White-noise Random Excitation	69
3.	Turbulent Boundary Layer Excitation	88
2.6	Response of Simply-supported Sandwich Beams having 'Elasto-dissipative' core to Random Excitation.	90
1.	Equation of Motion	90
2.	Transfer Function	94
3.	Response to Random Excitations	95
2.7	Results and Discussion-Beams with 'Elasto-dissipative' Core	95
1.	Introduction	95
2.	White-noise Random Excitation	96
3.	Turbulent Boundary Layer Excitation	98

CHAPTER 3	RESPONSE OF TWO-LAYER ELASTIC- VISCOELASTIC BEAMS TO RANDOM EXCITATION	100
3.1	Introduction and Assumptions	100
3.2	Equation of Motion of a Two-layer Elastic-Viscoelastic Beam	102
3.3	Response of a simply-supported 2-layer Elastic-Viscoelastic Beam to Random Excitation	105
1.	Transfer Function	105
2.	Response to Random Excitation	105
3.4	Results and Discussion	107
1.	Introduction	107
2.	White-noise Random Excitation	108
3.	Turbulent Boundary Layer Excitation	114
CHAPTER 4	RESPONSE OF THREE-LAYER SANDWICH PLATES TO RANDOM EXCITATION	121
4.1	Introduction	121
4.2	Assumptions	121
4.3	Equation of Motion of a 3-layer Sandwich Plate	123
4.4	Response of a 3-layer Viscoelastic Core Sandwich Plate, Simply-supported along the edges, to White-noise Random Excitation	133
1.	The Viscoelastic Model	133
2.	Equation of Motion	133
3.	Transfer Function of the System	135
4.	Response to White-noise Random Excitation of Distributed Load Type.	137

	5.	Response to White-noise Random Excitation of Point Load Type	142
4.5		Results and Discussion-Plates with Viscoelastic Core	148
	1.	Introduction	148
	2.	Response Characteristics	149
4.6		Response of a Sandwich Plate having 'Elasto-dissipative' Core and Simply Supported along the edges, to Random Excitation	159
	1.	Equation of Motion	159
	2.	Transfer Function	161
	3.	Response to White-noise Random Excitation	162
4.7		Results and Discussion - Plates with 'Elasto-dissipative' Core	162
	1.	Introduction	162
	2.	Response Characteristics	162
CHAPTER 5		RESPONSE OF TWO-LAYER ELASTIC - VISCOELASTIC PLATES TO RANDOM EXCITATION	167
	5.1	Introduction and Assumptions	167
	5.2	Equation of Motion of a Two-layer Elastic-viscoelastic Plate	167
	5.3	Response of a 2-layer Simply Supported Elastic-viscoelastic Plate to Random Excitation	172
	1.	Transfer Function	172
	2.	Response to Random Excitation	172
	5.4	Results and Discussion	173
	1.	Introduction	173

2.	Response Characteristics	173
CHAPTER 6	EXPERIMENTAL WORK	183
6.1	Introduction	183
6.2	Description of set-up	
	(a) White-noise Random Excitation	183
	(b) Turbulent Boundary Layer Excitation	185
6.3	Preparation of Specimens	188
6.4	Measurement Procedure	
	(a) White-noise Random Excitation	191
	(b) Turbulent Boundary Layer Excitation	195
6.5	Response Measurements	198
6.6	Theoretical Results	201
6.7	Comparison of Results and Discussion	204
CHAPTER 7	CONCLUSIONS, DISCUSSION AND FURTHER WORK	209
7.1	Introduction	209
7.2	General Discussion and Conclusions	
1.	Theoretical Results	209
2.	Experimental Work	213
3.	General Remarks	214
7.3	Further Work	214
REFERENCES	216
APPENDIX A	Four-element Viscoelastic Model	240
APPENDIX B	Random Vibration Analysis	262
APPENDIX C	Evaluation of Integrals	267
APPENDIX D	Determination of Elements of the Model for the Viscoelastic Material used in Experimental Test Specimens	272