

A THESIS ON
STABILITY OF PLANE POISEUILLE FLOW OF SLIGHTLY
ELASTICO-VISCOUS LIQUID

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Submitted to the Indian Institute of
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of the Degree of Doctor of Philosophy
in Mathematics
1972

CERTIFICATE

This is to certify that the thesis entitled "Stability of plane Poiseuille flow of a slightly elastico-viscous liquid", being submitted by Mrs. Roxna Swamy to the Indian Institute of Technology, Delhi, for the award of the degree of Doctor of Philosophy in Mathematics, is a record of bonafide research work carried out by her. She has worked under my guidance and supervision for the last two and a half years.

The thesis has reached the standard fulfilling the requirements of the regulations relating to the degree. The results obtained in this thesis have not been submitted in part or in full, to any other university or institute for the award of any degree of diploma.

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A C K N O W L E D G E M E N T S

I should like to express my gratitude to my thesis advisor Prof. M.P. Singh, Department of Mathematics, Indian Institute of Technology, Delhi, and to Prof. M.K. Jain, Head of the Department of Mathematics, Indian Institute of Technology, Delhi for their constant encouragement and interest in the progress of my work.

My thanks are also due to the authorities of the Indian Institute of Technology, Delhi for providing me with a scholarship and all the facilities of the Institute during my research work.

Finally I thank Shri D.R. Joshi for his work in typing the manuscript.

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