

# **FINITE ELEMENT STUDY OF GAS-SOLID FLOW IN VERTICAL PIPES**

by  
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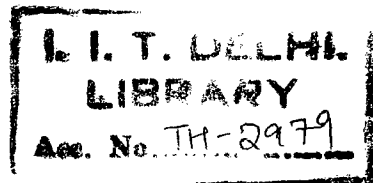
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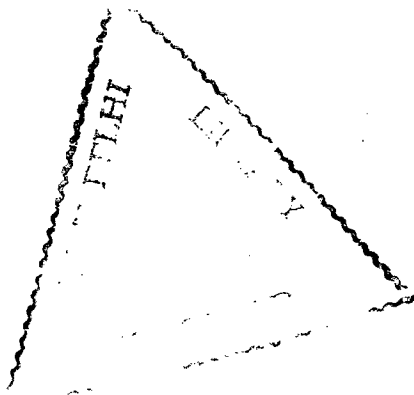
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Gas-Solid flow;  
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62



## CERTIFICATE

This is to certify that the thesis entitled **FINITE ELEMENT STUDY OF GAS-SOLID FLOW IN VERTICAL PIPES** being submitted by **Rajeev Jain** to the **Indian Institute of Technology, New Delhi (India)** for the award of the degree of Doctor of Philosophy in Applied Mechanics Department is a bonafide research work carried out by him under our supervision and guidance. The research reports and the results presented in this thesis have not been submitted in part or in full to any other University or Institute for the award of any degree or diploma.



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(RAJEEV JAIN)

## ABSTRACT

An Eulerian-Eulerian model is used to simulate gas solid flows in vertical pipes. In this model the carrier phase and the particulate phase are treated as two interacting and coexisting continua. Continuum equations for the phases derived by volume averaging of the instantaneous single-phase equations are used. The volume-averaged equations obtained are further time-averaged to obtain the mean equations for the turbulent flow. Both the gas and the particle phase are considered to be Newtonian fluids.

Two-equation  $k-\varepsilon$  turbulence model is employed for the carrier gas. The standard model for single-phase flow is modified for presence of particles in line with the formulation of Elghobashi and Abou-Arab (1983). The interaction due to drag was considered in the momentum equations of the phases.

The two-dimensional elliptic system of equations is solved by the finite element method. Weak Galerkin formulation is used. The primitive variable formulation is employed. A Picard-type iteration scheme is utilized to handle the non-linearity in the advection terms, average properties and transport coefficients of the phases. Eight-noded isoparametric elements are used. Pressure is approximated by bilinear interpolating functions. Wall functions are employed for both the phases to model the flow close to the wall. Option of upwinding is incorporated.

A computer code is developed based on this formulation. It is validated for cases of developing laminar and turbulent pipe flows. The results of two-phase case are compared with the experimental results of Tsuji *et al.* (1984) and are found to be in reasonable agreement.

Parametric studies are carried out to determine the effect of different variables on the flow quantities like the hold up, pressure drop, turbulence intensity, velocity and gas and particle viscosity. The significant variables of the problem and their ranges considered are pipe diameter from 30.5 to 100 mm, particle diameter from 100 to 600 microns, volumetric concentration from 0.25% to 2.0%. Numerical values of slip and pressure drop are compared with the correlations available in literature. A correlation for slip ratio is derived using multiple regression analysis. The solid friction factor is found to lie in the range 0.01 to 0.02.

The use of finite element code in design of vertical pipes is illustrated by means of a sample design problem.

A wear model is used to estimate the erosion of pipe from the computed two-phase flow field. The effect of the parameters on the wear rate is also studied. The wear rate increased as the cube of gas velocity and linearly with volumetric concentration of particles and decreased slightly with particle diameter.

A CFD code 'FLOW2P' is developed based on the Eulerian model using the finite element method, which will give the designer a tool for the design of vertical gas particle flow systems and enhance his confidence in predicting their performance.

# TABLE OF CONTENTS

|                                                     |           |
|-----------------------------------------------------|-----------|
| <b>Acknowledgements</b>                             | ii        |
| <b>Abstract</b>                                     | iv        |
| <b>Table of Contents</b>                            | vi        |
| <b>List of Figures</b>                              | x         |
| <b>List of Tables</b>                               | xv        |
| <b>Nomenclature</b>                                 | xvi       |
| <br>                                                |           |
| <b>CHAPTER 1          INTRODUCTION</b>              | <b>1</b>  |
| 1.1    Need For Numerical Modeling                  | 3         |
| 1.2    Modeling Approaches                          | 5         |
| 1.2.1   Lagrangian Models                           | 5         |
| 1.2.2   Eulerian Models                             | 8         |
| 1.2.3   Particic Turbulence Interaction             | 9         |
| 1.3    Wear                                         | 11        |
| 1.4    Present Study                                | 12        |
| 1.5    Objectives of the Present Study              | 15        |
| <br>                                                |           |
| <b>CHAPTER 2          MATHEMATICAL FORMULATION</b>  | <b>19</b> |
| 2.1    Averages and Averaging Theorems              | 20        |
| 2.2    Volume Average and Intrinsic Average         | 20        |
| 2.3    Minimum Volume for Volume Averaging          | 22        |
| 2.4    Volume Averaging Theorems                    | 23        |
| 2.5    Volume-Averaged Equations                    | 24        |
| 2.6    Volume-Time Averaged Equations               | 27        |
| 2.7    Constitutive Relations for the Stress Tensor | 30        |
| 2.8    Transport Coefficients                       | 31        |
| 2.9    Governing Equations for Vertical Pipe        | 31        |
| 2.10   Turbulence Model                             | 36        |
| 2.11   Transport Coefficients                       | 40        |

|                  |                                                                     |           |
|------------------|---------------------------------------------------------------------|-----------|
| 2.11.1           | Turbulent Diffusivity of the Dispersed Phase, $D_p$                 | 40        |
| 2.11.2           | Turbulent Kinematic Viscosity of the Dispersed Phase, $\nu_{pt}$    | 41        |
| 2.11.3           | Laminar Kinematic Viscosity of the Dispersed Phase, $\nu_p$         | 41        |
| 2.12             | Boundary Conditions                                                 | 42        |
| 2.12.1           | Centerline                                                          | 42        |
| 2.12.2           | Inlet                                                               | 42        |
| 2.12.3           | Exit                                                                | 43        |
| 2.12.4           | Wall                                                                | 43        |
| <b>CHAPTER 3</b> | <b>FINITE ELEMENT FORMULATION</b>                                   | <b>45</b> |
| 3.1              | Derivation of the Finite Element Equations                          | 45        |
| 3.1.1            | Continuity and Momentum Equations                                   | 45        |
| 3.1.2            | General Transport Equation                                          | 51        |
| 3.2              | The Finite Element                                                  | 53        |
| 3.3              | Shape (Interpolation) Functions                                     | 54        |
| 3.4              | Coordinate Transformation                                           | 56        |
| 3.5              | Numerical Integration                                               | 57        |
| 3.6              | The Solution Procedure                                              | 58        |
| 3.6.1            | Check for Convergence                                               | 60        |
| 3.6.2            | Solver                                                              | 60        |
| 3.7              | Upwinding                                                           | 61        |
| 3.8              | Boundary Conditions                                                 | 63        |
| 3.8.1            | Inlet                                                               | 63        |
| 3.8.2            | Outlet                                                              | 64        |
| 3.8.3            | Centerline                                                          | 64        |
| 3.8.4            | Wall                                                                | 64        |
| <b>CHAPTER 4</b> | <b>CODE VALIDATION AND COMPARISON<br/>WITH EXPERIMENTAL RESULTS</b> | <b>67</b> |
| 4.1              | Mesh Independence Studies                                           | 67        |
| 4.1.1            | Single-Phase Flow                                                   | 68        |
| 4.1.2            | Two-phase flow                                                      | 71        |
| 4.2              | Validation for Single-Phase Flow                                    | 73        |
| 4.2.1            | Developing pipe flow -- Laminar case                                | 73        |
| 4.2.2            | Developing pipe flow -- Turbulent case                              | 76        |

|                  |                                                      |            |
|------------------|------------------------------------------------------|------------|
| 4.3              | Comparison with Experimental Data for Two-Phase Flow | 78         |
| 4.4              | Comparison of Pressure Drop with Correlations        | 84         |
| <b>CHAPTER 5</b> | <b>RESULTS AND DISCUSSION</b>                        | <b>85</b>  |
| 5.1              | Parameters and their Ranges                          | 86         |
| 5.2              | The Slip or Holdup Effect                            | 87         |
| 5.2.1            | Correlations for Slip Ratio or Holdup Ratio          | 88         |
| 5.2.2            | Effect of Particle Diameter on Slip Ratio            | 91         |
| 5.2.3            | Effect of Concentration on Slip Ratio                | 93         |
| 5.2.4            | Effect of Pipe Diameter on Slip Ratio                | 95         |
| 5.2.5            | Effect of Particle Density on Slip Ratio             | 98         |
| 5.2.6            | Effect of Gas Density on Slip Ratio                  | 99         |
| 5.2.7            | Proposed Slip Ratio Correlation                      | 101        |
| 5.3              | Pressure Drop                                        | 102        |
| 5.3.1            | Effect of Particle Diameter                          | 104        |
| 5.3.2            | Effect of Volumetric Concentration                   | 106        |
| 5.3.3            | Effect of Pipe Diameter                              | 107        |
| 5.3.4            | Effect of Particle Velocity                          | 107        |
| 5.4              | Sample Design Problem                                | 110        |
| 5.5              | Turbulence                                           | 115        |
| 5.5.1            | Effect of Particle Diameter on Turbulence            | 116        |
| 5.5.2            | Effect of Volumetric Concentration on Turbulence     | 116        |
| 5.5.3            | Effect of Pipe Diameter on Turbulence                | 118        |
| 5.6              | Viscosity                                            | 119        |
| 5.6.1            | Effect of Particle Diameter                          | 119        |
| 5.5.2            | Effect of Volumetric Concentration                   | 121        |
| 5.5.3            | Effect of Pipe Diameter                              | 123        |
| 5.7              | Velocity                                             | 124        |
| <b>CHAPTER 6</b> | <b>WEAR</b>                                          | <b>127</b> |
| 6.1              | Effect of Gas Velocity                               | 128        |
| 6.2              | Effect of Concentration                              | 131        |
| 6.3              | Effect of Particle Diameter                          | 133        |
| 6.4              | Effect of Pipe Diameter                              | 136        |
| 6.5              | Sample Design Problem Revisited                      | 137        |

|                   |                                        |            |
|-------------------|----------------------------------------|------------|
| <b>CHAPTER 7</b>  | <b>CONCLUSIONS AND RECOMMENDATIONS</b> | <b>141</b> |
| 7.1               | Conclusions                            | 141        |
| 7.2               | Recommendations                        | 144        |
| <b>REFERENCES</b> |                                        | <b>145</b> |