

NONLINEAR WAVE PHENOMENA AND TRANSPORT  
PHENOMENA IN WEAKLY IONIZED MAGNETOPLASMAS

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## P R E F A C E

In recent years increasing attention has been given to the study of plasma dynamics and various processes occurring in plasmas. An important problem in this field is the study of the propagation of electromagnetic waves. A particular case of this is the behavior of a plasma in an electromagnetic field which is uniform in space but variable in time.

In equilibrium and in the absence of fields the plasma is described by the well known Maxwellian distribution function, namely

$$f = N \left( \frac{m}{2\pi kT} \right)^{3/2} e^{-\frac{mv^2}{2kT}} .$$

However in the presence of a variable field the function  $f$  is no more Maxwellian and in fact in a sufficiently strong field it is not even approximately so. The changed distribution function in this case is determined from the Boltzmann equation.

At high electromagnetic fields, the scattering processes in a plasma become field dependent and the response of the plasma becomes nonlinear. As a direct consequence, the phenomena of parametric amplification and mixed frequency generation appear. This finds direct

application in diagnostic techniques, frequency multipliers and parametric amplifiers.

Nonlinearities in plasmas may be due to either of the two independent mechanisms - the resistive or the reactive mechanism. In the reactive mechanism the nonlinearity is due to an electron density gradient induced by the applied electric field. In the propagation of longitudinal plasma waves it is the reactive mechanism which is responsible for longitudinal oscillations. In a magnetoactive plasma the reactive mechanism involves the nonlinear interaction of the magnetic field of the wave with the current density of electrons. The resistive nonlinearity is due to a change in electron velocity brought about by the field, which in turn causes changes in such parameters as the electron temperature and the collision frequency, so that electrodynamic processes in the plasma become nonlinear. Transverse harmonics are generated due to this mechanism; only odd harmonics are generated in the absence of a d.c. field. Throughout the thesis we have considered the resistive mechanism as the dominant one for wave mixing.

In a collisional plasma the electrons absorb energy from the field faster than they lose it to the heavy

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scatterers. This gives rise to heating of electrons; consequently the collision frequency becomes a function of the field. The amplitude of this oscillating component of electron temperature, given by<sup>1</sup>

$$\frac{e^2 E^2}{3mk(\omega^2 + \nu^2)} \left[ \frac{(\delta\nu^2 - 2\omega^2)}{4\omega^2 + \delta^2\nu^2} \cos 2\omega t \right. \\ \left. + \frac{\omega\nu(2+\delta)}{4\omega^2 + \delta^2\nu^2} \sin 2\omega t \right]$$

(where  $\delta$  is the fraction of energy transferred to a neutral particle by an electron during one collision,  $\omega$  is the incident wave frequency and  $\nu$  is the momentum transfer collision frequency), is greater for low frequency waves ( $\omega < \delta\nu$ ) than for high frequency waves ( $\omega > \delta\nu$ ). Therefore, we have investigated the generation of VLF waves as a result of nonlinear interaction of two microwaves with a plasma slab in the presence of a high d.c. field. We find that the reflected and transmitted VLF power is very high compared with the case of high frequency waves. In the particular case of a germanium semiconductor at 78°K and for moderately strong d.c. fields, the harmonic field is as much as 4% of the fundamental. The generated VLF power is found to be maximum for an optimum value of the nonlinearity parameter, which is a function

of carrier mass, collision frequency and mass of scatterer. The VLF power also exhibits maxima and minima at various slab thicknesses showing the interference effects between the forward and the backward propagating waves.

In the high frequency region, the nonlinear mixing of moderately strong microwaves does not result in high yields of harmonic power. According to calculations<sup>2,3,4</sup> based on the perturbation theory, the increase in amplitude of the fundamental field should result in a considerable increase in harmonic current density. Therefore we have studied the nonlinear mixing of two strong microwaves in an inert gas plasma. For strong collisions ( $\nu \approx \omega$ ) the generated harmonic and combination frequency current densities in a helium plasma show a maximum (for an optimum value of wave intensity) while a continuous increase is found for weak collisions ( $\nu \ll \omega$ ). In the presence of a static magnetic field, the generation of odd harmonics is not possible with only one mode of the fundamental wave though even harmonics can be generated if a d.c. field is present. Resonances appear at fundamental as well as at harmonic frequencies.

The propagation of strong microwaves in plasmas cannot be treated by a perturbation theory as the plasma parameters themselves become functions of the field. In such cases

an estimate of the reflected and transmitted fundamental and harmonic fields can be obtained numerically only. Accordingly we have carried out a numerical evaluation of the nonlinear Maxwell's equations by Runge-Kutta method, to determine the reflected and transmitted fields of the fundamental and generated third harmonic waves, when a high amplitude microwave interacts nonlinearly with a weakly ionized nitrogen plasma slab. For an incident field of  $\sim 3\epsilon_0$  and an incident wave frequency of  $\sim 10^{10}$  rad/sec, the peak power of the third harmonic obtained is  $\sim 0.04\%$  of the fundamental, which is in fairly good agreement with experiments. The harmonic fields also increase with electron concentration in the range  $\omega < \omega_p < 3\omega$  ( $\omega_p$  being the plasma frequency) and show resonant increase at fundamental and harmonic frequencies.

Apart from wave propagation another important aspect of plasma dynamics is the transport phenomena. A knowledge of the transport coefficients of plasmas is important in connection with the confinement of thermonuclear plasmas, for MHD power generators and for an understanding of the response of natural plasmas to electromagnetic signals. The transport phenomena for fully ionized plasmas both in the absence and in the presence of a magnetic field has already been studied extensively<sup>5,6,7</sup>. The presence

of a magnetic field affects the plasma in two ways depending on the field strength. Weak magnetic fields affect only the trajectories of the particles between collisions while strong magnetic fields affect the interparticle collisions also. A study of transport phenomena in weakly ionized gases in the presence of such strong magnetic fields is important in the E and F regions of the ionosphere, solar atmosphere, interstellar gas etc. and for some laboratory plasmas. We have therefore determined some transport coefficients for a homogeneous, weakly ionized Lorentzian plasma in the presence of a uniform external magnetic field which can affect collisions also. The electron-ion collisions are taken into account by means of the modified Fokker-Planck equation and the electron-neutral collision frequency,  $\nu$  is taken as velocity ( $u$ ) dependent or velocity independent, depending on the energy range under consideration. The variation of the transport coefficients with magnetic field is determined for  $\nu$  varying as  $u^s$  ( $s$  being a positive or negative integer) and we find that qualitatively the behavior does not change by changing  $s$ ; however for weak magnetic fields the magnitudes of these transport coefficients increase with the decrease in  $\nu$  or  $s$ , whereas for strong magnetic fields the transverse components of the transport coefficients

decrease and the Hall components tend to saturate with the decrease of  $\nu$  or  $s$ .

The above mentioned studies on wave propagation and transport phenomena have been carried out including only the elastic collisions (i.e. collisions in which the energy is conserved). However, depending on the electron energies involved we should include the effect of the various inelastic collisions like rotational, vibrational, recombination etc. Here we have considered the energy regime in which the rotational levels get excited but not the vibrational levels. The Boltzmann equation has been used to obtain an expression for the isotropic distribution function for electrons in a slightly ionized homogeneous plasma subjected to a microwave field. Inelastic collisions have been included on assuming that the average kinetic energy of the electrons is very much greater than the internal energy difference of the molecules for allowed transitions. Expressions for the current density and induced field in a nonlinear medium have also been obtained. The theory outlined is applied to study the phenomena of demodulation of an amplitude modulated wave. In the D-region of the ionosphere only rotational excitation of the nitrogen molecules is important; however, we find

that these inelastic collisions contribute negligibly to the current density as well as to conductivity.

Chapters I and II deal with the nonlinear mixing of two microwaves in the VLF and HF regions respectively. Evaluation of the third harmonic fields, generated by the nonlinear mixing of a high amplitude microwave with a plasma slab, has been carried out in chapter III. Chapter IV is devoted to the study of transport phenomena in weakly ionized magnetoplasmas. The effect of inelastic collisions on electron distribution function and wave propagation has been included in chapter V.

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