

Tree Spanners

Theory and Algorithms

By

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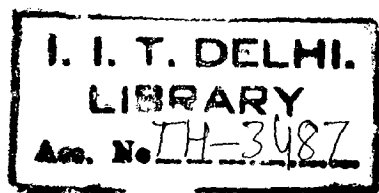
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Trees ② Algorithms



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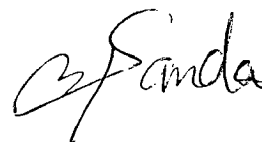
To
Maa & Baba

Certificate

This is to certify that the thesis entitled “**Tree Spanners: Theory and Algorithms**” submitted by “**Ms Anita Das**” to the Indian Institute of Technology Delhi, for the award of the Degree of Doctor of Philosophy, is a record of the original bona fide research work carried out by her under my supervision and guidance. The thesis has reached the standards fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted in part or full to any other university or institute for the award of any degree or diploma.

New Delhi
May 2007



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Abstract

A **tree t -spanner** T of a connected graph G is a spanning tree of G such that the distance between any two vertices in T is at most t times their distance in G . A graph that has a tree t -spanner is called a **tree t -spanner admissible graph**. While the problem of recognizing whether a graph is tree t -spanner admissible is NP-Complete for any fixed $t \geq 4$ and is polynomially solvable for $t \leq 2$, the case $t = 3$ is still open despite lot of efforts from various researchers over the last one decade.

Given an integer $t \geq 1$, the problems associated with tree t -spanner are:

- (1) **Characterization:** Characterize tree t -spanner admissible graphs.
- (2) **Recognition:** Recognize whether a given graph is tree t -spanner admissible.
- (3) **Construction:** Construct a tree t -spanner of a given tree t -spanner admissible graph.

Though the above three problems concerning tree 3-spanner are open for general graphs, it has been shown by various researchers that these problems can be solved in polynomial time for various graph classes with special structures. However, most of the graph classes for which the tree 3-spanner problems have been solved admit tree 3-spanners, i.e., every graph in the graph class admits a tree 3-spanner. So, problem (1) and problem (2) turn out to be trivial for these graph classes.

This thesis deals with the above three problems concerning tree t -spanner for $t = 3$ and $t = 4$.

H.-O. Le and V. B. Le (1999, Networks, 34(2), 81-87) have claimed that directed path graphs always admit tree 3-spanners. However, it has been shown in this thesis that this result is incorrect by producing a directed path graph that does not admit a tree 3-spanner. So, the tree 3-spanner problems are now open for directed path graphs.

In this thesis, the tree 3-spanner problems have been solved for the following graph classes.

- 2-sep directed path graphs.
- Directed path graphs of diameter at most three.
- 2-sep chordal graphs.
- Simple chordal bipartite graphs.

It has been shown in this thesis that none of the above classes of graphs except 2-sep chordal graphs admit tree 3-spanners in general. It is known that there are 2-sep chordal graphs that do not admit tree 3-spanners. So, characterization and recognition of tree 3-spanner admissible graphs of the above classes become very interesting. The emphasis of this thesis is to propose good structural characterization that leads to efficient recognition algorithm.

The tree 4-spanner problems in 2-trees have also been solved in this thesis.

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