Tree Spanners

Theory and Algorithms

By

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Submitted in fulfillment of the requirements
of the degree of Doctor of Philosophy

to the

Indian Institute of Technology Delhi
May 2007

Trees & Algorithms

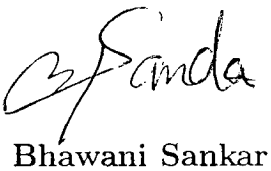
To

Maa & Baba

Certificate

This is to certify that the thesis entitled **"Tree Spanners: Theory and Algorithms"** submitted by **"Ms Anita Das"** to the Indian Institute of Technology Delhi, for the award of the Degree of Doctor of Philosophy, is a record of the original bona fide research work carried out by her under my supervision and guidance. The thesis has reached the standards fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted in part or full to any other university or institute for the award of any degree or diploma.



New Delhi
May 2007

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Acknowledgements

It gives me immense pleasure to express my deepest gratitude to my thesis supervisor Dr. B. S. Panda, for his expert guidance, constant inspiration, and all his help for carrying out this work. This research would have not been possible without his careful guidance and invaluable suggestions. The discussions with him were very much enlightening and motivated me to work with greater zeal and enthusiasm. I am indebted to him for his patience and support. It is my privilege to have the opportunity to work under his supervision.

I am thankful to IIT Delhi authorities for providing me the financial assistance by awarding me a GATE research scholarship during the period of my study and research. I express my regards to Prof. B. Chandra, Head of the Department, for her support and affection. Special thanks to my SRC (Student Research Committee) members: Dr. Niladri Chatterjee and Prof. S. K. Gupta for their valuable suggestions. I express my gratitude to all the faculty members of The Department of Mathematics, IIT Delhi, for their encouragement.

I am quite fortunate to have friends like Krishna, Manju, Sarita, and Shailly. Thanks for all the care, love, understanding, and encouragement. I spent a great time with them. Special thanks to Krishna from whom I learnt a lot in many aspect. I know, I cannot escape just by saying thanks to them.

I would like to thank Preeti, Dinu, and Bighnaraj for their encouragement and cooperation. Special thanks to Dinu for spending his valuable time for careful reading of an early draft of this thesis.

It is my pleasure to thank my senior colleagues Deepa, Sonia, Prabhakar, and Vidyottama for their suggestions and help. Also, Pratibha, Kanchan, Megha, Reshma, Vaneeta, Geeta, Deepali, Anulekha, Vandana, Pooja, Pandey ji, Piyush, Mukesh, Sachin, Dhirendra, Prajapati, and Puneet are thanked for providing me a great company while doing research. Thanks to Smruti, Arya, Purnendu, Sanjay, Sasmita,

Sarala, and Neeru for their cooperation in many ways.

I am thankful to Raju for his cooperation and great help during the initial days in IIT. Thanks for all the support and courage you have given.

I am sincerely grateful to my extended family. A great debt of gratitude to my parents, brother Bidya and sister Mama for their love, affection, encouragement, understanding, unfailing faith, and lot more.

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Abstract

A **tree t-spanner** T of a connected graph G is a spanning tree of G such that the distance between any two vertices in T is at most t times their distance in G. A graph that has a tree t-spanner is called a **tree t-spanner admissible graph**. While the problem of recognizing whether a graph is tree t-spanner admissible is NP-Complete for any fixed $t \geq 4$ and is polynomially solvable for $t \leq 2$, the case $t = 3$ is still open despite lot of efforts from various researchers over the last one decade.

Given an integer $t \geq 1$, the problems associated with tree t-spanner are:

(1) **Characterization:** Characterize tree t-spanner admissible graphs.

(2) **Recognition:** Recognize whether a given graph is tree t-spanner admissible.

(3) **Construction:** Construct a tree t-spanner of a given tree t-spanner admissible graph.

Though the above three problems concerning tree 3-spanner are open for general graphs, it has been shown by various researchers that these problems can be solved in polynomial time for various graph classes with special structures. However, most of the graph classes for which the tree 3-spanner problems have been solved admit tree 3-spanners, i.e., every graph in the graph class admits a tree 3-spanner. So, problem (1) and problem (2) turn out to be trivial for these graph classes.

This thesis deals with the above three problems concerning tree t-spanner for $t = 3$ and $t = 4$.

H.-O. Le and V. B. Le (1999, Networks, 34(2), 81-87) have claimed that directed path graphs always admit tree 3-spanners. However, it has been shown in this thesis that this result is incorrect by producing a directed path graph that does not admit a tree 3-spanner. So, the tree 3-spanner problems are now open for directed path graphs.

In this thesis, the tree 3-spanner problems have been solved for the following graph classes.

- 2-sep directed path graphs.
- Directed path graphs of diameter at most three.
- 2-sep chordal graphs.
- Simple chordal bipartite graphs.

It has been shown in this thesis that none of the above classes of graphs except 2-sep chordal graphs admit tree 3-spanners in general. It is known that there are 2-sep chordal graphs that do not admit tree 3-spanners. So, characterization and recognition of tree 3-spanner admissible graphs of the above classes become very interesting. The emphasis of this thesis is to propose good structural characterization that leads to efficient recognition algorithm.

The tree 4-spanner problems in 2-trees have also been solved in this thesis.

Contents