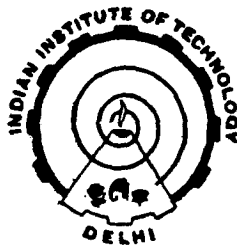


**KINETO-ELASTODYNAMIC ANALYSIS  
OF  
PLANAR FOUR-BAR MECHANISMS**

**I.I.T. LIBRARY DELHI**

BY  
**KESHAV CHATURBHAI MAKWANA**

A THESIS SUBMITTED IN  
FULFILMENT OF THE REQUIREMENT  
FOR THE AWARD OF THE DEGREE  
DOCTOR OF PHILOSOPHY



DEPARTMENT OF MECHANICAL ENGINEERING  
INDIAN INSTITUTE OF TECHNOLOGY, DELHI  
NEW DELHI - 110016  
INDIA  
SEPTEMBER 1990

DEDICATED

TO

MY BELOVED PARENTS

SHRI CHATURBHAI BUTABHAI MAKWANA

and

SHRIMATI BALUBEN CHATURBHAI MAKWANA

Whose Inspiration, constant moral Encouragement and  
Blessings enabled me to ultimately  
complete this research successfully

and

MY RESPECTED TEACHERS

Whose moral Support and Encouragement  
made this research really possible

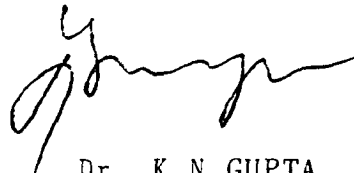
and

PROFESSOR KAILASHNATH GUPTA

Whose valuable guidance and kind help  
really made this research successful

## CERTIFICATE

This is to certify that the thesis entitled "KINETO-ELASTODYNAMIC ANALYSIS OF PLANAR FOUR-BAR MECHANISMS" being submitted by shri Keshav Chaturbhai Makwana to the Indian Institute of Technology, Delhi, for the award of the degree of 'Doctor of Philosophy' in Mechanical Engineering, is a record of bonafide research work carried out by him. He has worked under my guidance and supervision and has fulfilled the requirements for the submission of this thesis which has attained the standard required for the Ph.D degree of the institute. The subject matter embodied in this thesis has not been submitted in part or full elsewhere for the award of any degree or diploma.



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## ABSTRACT

The present thesis is an attempt at developing a methodology for modelling and analysis of the planar four-bar mechanisms for studying the influence of KED (KINETO-ELASTODYNAMIC) effects on their performance. Such mechanisms are widely used in various types of machines. The accelerated pace of present design environment implies to have mechanisms having lighter weight components and operating at higher speeds without affecting their performance characteristics, stress and noise levels, power consumption and reliability. Therefore, it is absolutely essential to obtain a better understanding of the KED phenomenon under this situation. Because operation in high speed mode creates numerous problems, as KED phenomenon significantly affects the system performance and results in vibration, acoustic radiation, unnecessary wear and earlier fatigue failure, excessive elastic deformation, positioning inaccuracies at some critical portion of the system which sharply limits the operational speed and even reduces prescribed capacity of the system.

In the present work a crank-rocker mechanism having both coupler and rocker flexible has been investigated under two situations. In the first situation the crank is assumed to rotate at constant speed, while in the second situation it is free to have speed (variable) depending upon the inertia

and torque being transmitted. These studies form chapters 3 and 4 of the thesis. In both cases mathematical models have been constructed with the help of Hamilton's integral and method of Kantorovich. The models consist of ordinary second order differential equations. These are used to analyse the KED behaviour of unbalanced four-bar mechanisms. For seeking their solution, they are transformed into a standard form suitable for numerical integration by the RKM method and the Runge-Kutta-Verner method of fifth and sixth order. The solutions are then used to determine various performance parameters.

To carry out all these mathematical operations, two computer softwares are developed, one for response of elastic links when crank is constrained to have constant angular velocity and the other when it is free to have variable speed. The programmes are validated and established with reference to some cases for which results are available in the literature.

The effects of various parameters like input speed, damping and starting angle for initial condition, crank inertia on performance have been studied for two unbalanced example mechanisms. Sample results are given for the models in form of graphs and discussed.

Finally at the end, based on the investigations and results reported in different chapters, it is attempted to highlight major achievements, to elaborate limitations of the research pursued and to report main conclusions and scope of further research in the related area. The thesis concludes with a detailed list of references and appendices to support the main text of this dissertation.

CERTIFICATE  
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