

**STUDY OF ELECTROMAGNETIC WAVES ON A
STRUCTURED PERFECT ELECTRIC
CONDUCTOR (PEC) SURFACE**

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**DEPARTMENT OF ELECTRICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY DELHI**

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CONDUCTOR (PEC) SURFACE**

by

SAYAK BHATTACHARYA

DEPARTMENT OF ELECTRICAL ENGINEERING

Submitted

in fulfillment of the requirements of the degree of Doctor of Philosophy



INDIAN INSTITUTE OF TECHNOLOGY DELHI

JULY 2016

Certificate

This is to certify that the thesis titled **Study of Electromagnetic Waves on a Structured Perfect Electric Conductor (PEC) Surface** being submitted by **Mr. Sayak Bhattacharya** for the award of **Doctor of Philosophy** in **Department of Electrical Engineering** is a record of bona-fide work carried out by him under my guidance and supervision at the **Department of Electrical Engineering, Indian Institute of Technology Delhi**. The work presented in this thesis has not been submitted elsewhere, either in part or full, for the award of any other degree or diploma.

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It was 30th March, 2012, 10 : 45 a.m. I got the very first mail from Dr. Kushal Shah who showed interest in my profile. A very complicated chain of events led to this connection, a chain of events which broke my confidence into pieces but also led me to the day when our paths crossed. Since then, Kushal has always been the light I followed and eventually saw this day when I sit down and type in this acknowledgement part of the dissertation. I am fortunate and glad to be supervised by a wonderful person like him. Just saying thanks to Kushal is not enough for the intellectually stimulating discussion sessions, guidance and friendship (I would like to mention here, it was the first Teacher's Day during my stay at IIT Delhi and I wished him. His prompt answer: Come on, wish me on the Friendship Day)!

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“Behind the mountain, there are mountains”–Thanks to Mother Nature for creating the beautiful mountains which whisper in my ears, “Keep going”.

Sayak Bhattacharya

Whose woods these are I think I know.

His house is in the village though;

He will not see me stopping here

To watch his woods fill up with snow.

My little horse must think it queer

To stop without a farmhouse near

Between the woods and frozen lake

The darkest evening of the year.

He gives his harness bells a shake

To ask if there is some mistake.

The only other sound's the sweep

Of easy wind and downy flake.

The woods are lovely, dark and deep,

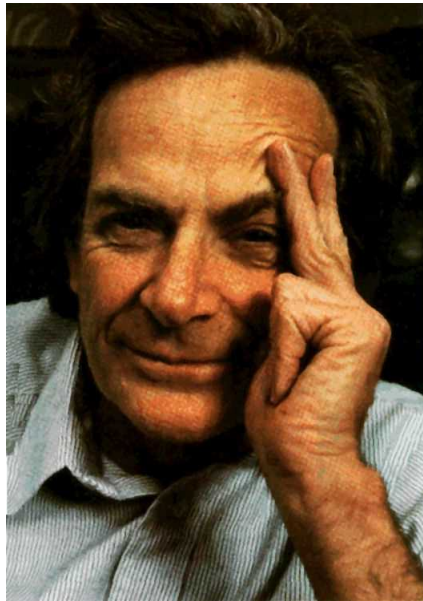
But I have promises to keep,

And miles to go before I sleep,

And miles to go before I sleep.

...Robert Frost (Stopping by Woods on a Snowy Evening)

*This thesis is dedicated to the adventures of the curious
character...*



Richard P. Feynman (1918 – 1988)

You will continue to be my inspiration.

Abstract

The low frequency behavior of spoof surface plasmon (SSP) is usually modeled by means of a close analogy with conventional surface plasmon polariton (SPP). In this limit, the SSP dispersion relation is derived based on an effective medium approach and is valid only when the operating wavelength λ is large as compared to the feature-size b and lattice constant a of the structure. However, experimental realization of SSP falls in a regime where the feature size of the structure is comparable to the wavelength and in this limit, the analogy between SPP and SSP breaks down.

In this dissertation, through finite difference time domain (FDTD) simulations, we show that in contrast to SPP propagation, SSP involves both TM and TE modes. Using a combination of angular spectrum representation and vector nature of the electromagnetic fields, we show that the near-field of SSP is described by multiple exponential decays instead of the single exponential which characterizes SPP. Further, we also show that existence of multiple modes is closely related to the anisotropic propagation which prominently manifests in the phenomenon of self-collimation (SC). In order to describe SC, we first describe the dispersion relation of SSP by the tight-binding model (TBM). With the help of this semi-analytical model, we propose a computationally efficient scheme for band-diagram calculation of SSP. This model is general enough to be applied to both metallic and dielectric photonic crystals and is valid even when the master equation of the photonic crystal is non-hermitian. The proposed method relies on calculation of eigenfrequencies only at the 3 high-symmetry points (in case of a 2D lattice) while rest of the band surface over the entire Brillouin zone and EFC are obtained analytically. Using this analytical expression of the band-structure, we are able to predict the SC frequency of SSP and centimeter-scale super collimation frequency in dielectric structure with an accuracy of 0.36% and 0.04%, respectively, as compared to numerical and experimental observations. In addition, we investigate the extraordinary transmission of microwaves and propose a structure which has a Q value 35 times better than that existing in the literature. This structure can be extremely useful for designing microwave notch filters.

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