

**ESTIMATION IN SOME NONREGULAR MODELS :
WITH OR WITHOUT RESTRICTIONS
ON PARAMETERS**

By

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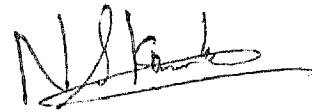
CERTIFICATE

This is to certify that the thesis entitled " Estimation in some nonregular models: with or without restrictions on parameters " which is being submitted by Vandurangi Ravindranath for the award of DOCTOR OF PHILOSOPHY (MATHEMATICS) to the Indian Institute of Technology, Delhi, is a record of bonafide research work carried out by him under our guidance and supervision. The thesis has reached the standards fulfilling the requirements of the regulations relating to the degree. The results obtained in the thesis have not been submitted to any other University or Institute for the award of any degree or diploma.



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To my beloved parents

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ABSTRACT

Simultaneous estimation problem of smaller and the larger of the parameters of the nonregular densities of form Hogg and Craig (1956) has been studied when the available samples are complete or type II censored. Theorems for the nonexistence of the unbiased estimators are proved and for those cases an alternate approach is suggested to get the estimators. The expressions for the estimators are then specialized to get new estimators for some well known population densities and the new estimators for uniform and exponential populations are compared in terms of bias, MSE and PN criteria with the available estimators in the literature. For Pareto distribution the estimators for the smaller and larger parameters are not available and hence some classes of scale equivariant estimators are considered and among them admissible subclasses of estimators in terms of MSE and absolute bias wise optimum estimators are found. Further marginal MLE and its risks are evaluated. Next, UMVU estimation of u-estimable functions of parameters of k nonregular populations is studied. Rohatgi and Selvavel (1993) single outlier problem is treated as mixture of several nonregular populations and the UMVUE for the common nonregular parameter of the k populations is obtained. Several illustrations are provided. Finally estimation of the location parameter when it is restricted by a known constant and also when it is bounded above by a location parameter of another population is studied and several new estimator are proposed. The new estimators are improvement over the existing estimators in terms of MSE and PN criteria but in some close neighborhood of the upper bounds of the parameters.

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