

DYNAMIC NONLINEAR CHARACTERISTICS OF SOIL-PILE SYSTEMS UNDER MACHINE INDUCED VIBRATIONS

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DYNAMIC NONLINEAR CHARACTERISTICS OF SOIL-PILE SYSTEMS UNDER MACHINE INDUCED VIBRATIONS

by

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to the



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*Dedicated to my family for their
love, endless support and
encouragement*

CERTIFICATE

This is to certify that the thesis entitled “**Dynamic Nonlinear Characteristics of Soil-Pile Systems under Machine Induced Vibrations**”, submitted by **Mr. Shiva Shankar Choudhary** to the **Indian Institute of Technology Delhi**, is a record of bonafide research work carried out by him under my supervision and is worthy of consideration for the award of **Doctor of Philosophy** degree. The research reported in this thesis has not been submitted in part or full to any other university or Institute for the award of any degree or diploma.

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ABSTRACT

In the present study, dynamic response characteristics of hollow steel piles under varying levels of rotating machine induced vertical and coupled (horizontal and rocking) excitations are investigated. A single pile and four set of pile groups (3-pile group, 2×2 , 2×3 , and 3×3 -pile group) are constructed in the field for dynamic testing. The frequency-amplitude responses of single and group piles are measured for four different eccentric moments and three different static loads under vertical and coupled vibration. The main purpose of the study is to monitor the effect of various influencing factors i.e., eccentric moment, static load and different pile arrangements on the dynamic response of piles under vertical and coupled vibration.

Theoretical study is performed using continuum approach with superposition method to determine the dynamic response of the single pile and pile groups under vertical and coupled modes of vibration. Both linear and nonlinear analyses are performed to find out the dynamic responses of the pile foundations. In linear analysis, a perfect bonding between the pile and soil is considered. However in the case of nonlinear analysis, weak boundary zones around the pile and soil-pile separation are incorporated. The theoretical responses obtained using both linear and nonlinear analyses are compared with the measured dynamic responses of piles. The linear analysis overestimated the resonant frequency and underestimated resonant amplitude values as compared to dynamic field test results for all the soil-pile systems. On the other hand, reasonable match between the test and nonlinear theoretical responses are found for single and group piles under different modes of vibration. In addition to that the possible variations of the weak boundary zone parameters and soil-pile separation lengths are predicted from the nonlinear analysis for vertical and coupled vibration. The variations of stiffness and

damping of soil-pile systems with frequency are presented using linear and nonlinear analysis. The dynamic pile-soil-pile interaction behaviors in terms of group efficiency ratio (GER) are also studied for different modes of vibration. Finally, a methodology is explained to convert the experimental results of the model pile to full scale prototype pile using scaling law.

Keywords: Coupled excitation, Damping, Group pile, Linear analysis, Nonlinear analysis, Separation, Scaling law, Single pile, Stiffness, Vertical vibration

सार

वर्तमान अध्ययन में, घुमावदार मशीन प्रेरित लंबवत और युग्मित (क्षैतिज और रॉकिंग) उत्तेजनाओं के विभिन्न स्तरों के तहत खोखले स्टील ढेर की गतिशील प्रतिक्रिया विशेषताओं की जांच की जाती है। गतिशील परीक्षण के लिए क्षेत्र में एक एकल पाइल और पाइल समूहों के चार सेट (3-पाइल समूह, 2×2 , 2×3 , और 3×3 -पाइल समूह) का निर्माण किया जाता है। एकल और समूह पाइल की आवृत्ति-आयाम प्रतिक्रिया चार अलग-अलग इक'सेन्ट्रिक् मअउमन्ट और लंबवत और युग्मित कंपन के तहत तीन अलग-अलग स्थैतिक भारों के लिए मापा जाता है। अध्ययन का मुख्य उद्देश्य लंबवत और युग्मित कंपन के तहत पाइल की गतिशील प्रतिक्रिया पर विभिन्न प्रभावकारी कारकों यानी इक'सेन्ट्रिक् मअउमन्ट, स्थैतिक भार और विभिन्न पाइल व्यवस्था के प्रभाव की निगरानी करना है।

सैद्धांतिक अध्ययन को कंपन के लंबवत और युग्मित मोड के तहत एकल पाइल और पाइल समूहों की गतिशील प्रतिक्रिया निर्धारित करने के लिए सुपरपॉजिशन विधि के साथ निरंतर दृष्टिकोण का उपयोग करके किया जाता है। पाइल फ़ाउन्डेशन के गतिशील प्रतिक्रियाओं को जानने के लिए दोनों लिनीअ और नॉन-लिनीअ विश्लेषण कर रहे हैं। लिनीअ विश्लेषण में, पाइल और मिट्टी के बीच एक आदर्श बंधन माना जाता है। हालांकि नॉन-लिनीअ विश्लेषण के मामले में, पाइल के चारों ओर कमजोर सीमा जोन और मिट्टी-पाइल अलगाव शामिल हैं। लिनीअ और नॉन-लिनीअ विश्लेषण दोनों का उपयोग कर प्राप्त सैद्धांतिक प्रतिक्रियाओं की तुलना पाइल के मापा गतिशील प्रतिक्रियाओं के साथ की जाती है। लिनीअ विश्लेषण ने सभी मिट्टी-पाइल प्रणालियों के लिए गतिशील क्षेत्र परीक्षण परिणामों की तुलना में अनुनासिक आवृत्ति और कम अनुमानित अनुनाद आयाम मानों को अधिक महत्व दिया। दूसरी ओर, परीक्षण और नॉन-लिनीअ सैद्धांतिक प्रतिक्रियाओं के बीच उचित मिलान कंपन के विभिन्न तरीकों के तहत एकल और समूह पाइल के लिए पाए जाते हैं। इसके अलावा कमजोर सीमा क्षेत्र पैरामीटर और मिट्टी-पाइल अलगाव की लंबाई के संभावित बदलावों को लंबवत और युग्मित कंपन के लिए नॉन-लिनीअ विश्लेषण से भविष्यवाणी की जाती है। आवृत्ति के साथ

मिट्टी-पाइल प्रणालियों की स्टिफ़नेस् और डैम्पिङ् की भिन्नताओं को लिनीअ और नॉन-लिनीअ विश्लेषण का उपयोग करके प्रस्तुत किया जाता है। समूह दक्षता अनुपात (जीईआर) के संदर्भ में गतिशील पाइल-मिट्टी-पाइल परस्पर व्यवहार का भी कंपन के विभिन्न तरीकों के लिए अध्ययन किया जाता है। अंत में, स्केलिंग कानून का उपयोग करके मॉडल पाइल के प्रयोगात्मक परिणामों को पूर्ण पैमाने पर प्रोटोटाइप डेर में बदलने के लिए एक पद्धति को समझाया गया है।

कीवर्ड: युग्मित उत्तेजना, डैम्पिङ्, समूह पाइल, लिनीअ विश्लेषण, नॉन-लिनीअ विश्लेषण, अलगाव, स्केलिंग कानून, एकल पाइल, स्टिफ़नेस्, लंबवत कंपन

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LIST OF SYMBOLS

Most of the symbols are defined as they appear in the thesis. Some of the most common symbols, which are used repeatedly, listed below.

Roman symbols

a_o	Dimensionless frequency
$\overline{a_o}$	Dimensionless frequency for composite medium
A	Cross-sectional area of the pile
A_1	Area of the topmost element of the pile
A_H	Amplitude of horizontal motion
A_{H1-res}, A_{H2-res}	First and second resonant amplitudes for horizontal motion
A_s	Steady state amplitude
A_V	Amplitude of vertical motion
A_{V-res}	Resonant amplitude for vertical motion
b	Pile radius plus thickness of inner soil medium ($= R + t_m$)
c	Coefficient of pile internal damping
c_u^c	Horizontal damping of embedded footing or pile cap
c_{uu}^1	Horizontal damping constant of single pile
$c_{u\psi}^c$	Cross damping of embedded footing or pile cap
$c_{u\psi}^1 = c_{\psi u}^1$	Cross damping constant of single pile
c_w^c	Vertical damping of embedded footing or pile cap
c_{ww}^1	Vertical damping constant of single pile
c_{ψ}^c	Rocking damping of embedded footing or pile cap
$c_{\psi\psi}^1$	Rocking damping constant of single pile
C_{cu1}, C_{cu2}	Stiffness and damping parameters of half-space for horizontal vibration
C_{cw1}, C_{cw2}	Stiffness and damping parameters of half-space for vertical vibration
$C_{c\psi1}, C_{c\psi2}$	Stiffness and damping parameters of half-space for rocking vibration
C_{uu}	Total horizontal damping constants of pile

List of Symbols

$C_{u\psi}$	Total cross damping constants of pile
$C_{\psi\psi}$	Total rocking damping constants of pile
d	Diameter of the pile
D	Damping ratio
D_i	Damping ratio corresponding to frequency of vibration ω_i
D_s	Dimensionless damping constant
D_{sm}	Dimensionless damping constant of inner medium
e	Eccentric distance of the rotating masses
E_b	Young's modulus of the bearing stratum
E_p	Young's modulus of the pile
E_s	Young's modulus of the soil along the pile
f_{c1}, f_{c2}	Dimensionless cross stiffness and damping parameters
f_{mc1}, f_{mc2}	Dimensionless cross stiffness and damping constants of the composite medium
f_{mu1}, f_{mu2}	Dimensionless horizontal stiffness and damping constants of the composite medium
f_{mw1}, f_{mw2}	Dimensionless vertical stiffness and damping constants of the composite medium
$f_{m\psi1}, f_{m\psi2}$	Dimensionless rocking stiffness and damping constants of the composite medium
f_n	Resonant frequency for vertical vibration
f_{n1}, f_{n2}	First and second resonant frequencies for coupled vibration
f_{u1}, f_{u2}	Dimensionless horizontal stiffness and damping parameters
f_{w1}, f_{w2}	Dimensionless vertical stiffness and damping parameters
$f_{\psi1}, f_{\psi2}$	Dimensionless rocking stiffness and damping parameters
f_{st}	Static flexibility of a single pile
$[f]$	Group flexibility matrix
$F(A)$	Restoring force
F_i	Dimensionless function
g	Acceleration due to gravity
G	Shear modulus of soil
G'	Imaginary part of the complex shear modulus of soil

G^*	Complex shear modulus of outer soil medium
G_{cb}	Dynamic shear modulus of soil below the footing
G_{cs}	Dynamic shear modulus of the side layers of footing
G_m	Shear modulus of inner soil medium
G_m^*	Complex shear modulus of inner weak soil medium
h	Embedded depth of footing or pile cap
H	Height (length) of the element
H_1, H_2	Horizontal end forces of element corresponding to end displacements
i	Complex number ($i = \sqrt{-1}$)
I	Moment of inertia of pile cross section
I_1	Moment of inertia of the topmost element of the pile
I_ψ	Mass moment of inertia about a horizontal axis passing through the center of gravity
k_{um}	Horizontal stiffness of the composite medium
k_{us}	Complex horizontal stiffness of soil per unit length of pile
k_{ut}	Complex horizontal stiffness of soil at the pile tip
k_{wm}	Vertical stiffness of the composite medium
k_{ws}	Complex vertical stiffness of soil per unit length of pile
k_{wt}	Complex vertical stiffness of soil at the pile tip
$k_{\psi m}$	Rocking stiffness of the composite medium
$k_{\psi s}$	Complex rocking stiffness of soil per unit length of pile
$k_{\psi t}$	Complex rocking stiffness of soil at the pile tip
$\bar{k}_u$	Horizontal static stiffness of single pile
$k_u^{(1)}$	The soil stiffness of the outer medium for horizontal vibration
$k_u^{(2)}$	The soil stiffness of the inner medium for horizontal vibration
k_u^c	Horizontal stiffness of embedded footing or pile cap
k_{uu}^1	Horizontal stiffness of single pile
$k_{u\psi}^1 = k_{\psi u}^1$	Cross stiffness of single pile
$k_{u\psi}^c$	Cross stiffness of embedded footing or pile cap

List of Symbols

$\bar{k}_w$	Vertical static stiffness of single pile
k_w^c	Vertical stiffness of embedded footing or pile cap
k_{ww}^1	Vertical stiffness of single pile
k_{ψ}^c	Rocking stiffness of embedded footing or pile cap
$k_{\psi\psi}^1$	Rocking stiffness of single pile
K_0	Modified Bessel functions of order zero
K_1	Modified Bessel functions of order one
K^G	Complex group stiffness
$K_e(A)$	Equivalent linear stiffness
K_{ij}	Elements of the structure matrix
K_{uu}	Total horizontal stiffness of pile
$K_{u\psi}$	Total cross stiffness of pile
K_{ww}	Total vertical stiffness of pile
$K_{\psi\psi}$	Total rocking stiffness of pile
K_{mu}^1	Total complex horizontal stiffness of single pile in composite medium
K_{mw}^1	Total complex vertical stiffness of single pile in composite medium
$K_{m\psi}^1$	Total complex rocking stiffness of single pile in composite medium
$K_{mu\psi}^1 = K_{m\psi u}^1$	Total complex cross stiffness of single pile in composite medium
K_{uu}^1	Complete horizontal stiffness of single pile
$K_{u\psi}^1 = K_{\psi u}^1$	Complete cross stiffness of single pile
K_{ww}^1	Complete vertical stiffness of single pile
$K_{\psi\psi}^1$	Complete rocking stiffness of single pile
K_u^G	Horizontal stiffness of pile group
$K_{u\psi}^G$	Cross stiffness of pile group
K_w^G	Vertical stiffness of pile group

K_{ψ}^G	Rocking stiffness of pile group
$[k_t]$	Element stiffness matrix of the soil underlying the pile tip
$[k_u]$	Element stiffness matrix for horizontal vibration
$[k_w]$	Element stiffness matrix for vertical vibration
$[K]$	Structure stiffness matrix
$[K_u]$	Structure stiffness matrix of single pile for horizontal vibration
$[K_w]$	Structure stiffness matrix of single pile for vertical vibration
l_s	Separation length between pile and soil
L	Length of the pile
m	Mass of eccentric rotating part in oscillator
m_{eff}	Effective mass of the system
m_s	Total mass of footing or pile cap
M	Real amplitude of moment
M_1, M_2	Moment of element corresponding to end rotations
$M(t)$	Total moment of excitation for coupled vibration
$M_e(t)$	Excitation moment
n	Number of piles in a group
N	SPT value
N_1, N_2	Vertical end forces of element corresponding to end displacements
N_{st}	Static axial force (load of the pile)
$N_x(t)$	Resultant horizontal reaction acting on embedded surface (sides) of footing
$N_{\psi}(t)$	Reactive moment of forces acting on footing sides about center of gravity
p	Lateral soil resistance
p_u	Ultimate soil resistance
P	Real amplitude of dynamic force
$P(t)$	Horizontal excitation force
$\{P\}$	Vector of forces

List of Symbols

r_1	Radius of the topmost element of the pile
r_c	Radius of footing or pile cap
R	Radius of circular pile
R_{inter}	strength reduction factor
$R_x(t)$	Horizontal reaction at the footing base
$R_\psi(t)$	Reactive moment of forces acting at footing base about center of gravity
s	Spacing of piles in a group
s_u	Undrained shear strength
S_{cu1}, S_{cu2}	Stiffness and damping parameters of side layer for horizontal vibration
S_{cw1}, S_{cw2}	Stiffness and damping parameters of side layer for vertical vibration
$S_{c\psi1}, S_{c\psi2}$	Stiffness and damping parameters of side layer for rocking vibration
S_{ou1}, S_{ou2}	Horizontal stiffness and damping parameters of outer medium
S_{mu1}, S_{mu2}	Dimensionless stiffness and damping parameters of composite medium for horizontal vibration
S_{mw1}, S_{mw2}	Dimensionless stiffness and damping parameters of composite medium for vertical vibration
$S_{m\psi1}, S_{m\psi2}$	Dimensionless stiffness and damping parameters of composite medium for rocking
S_{u1}, S_{u2}	Real and imaginary parts of the dimensionless complex stiffness in horizontal direction (u)
S_{w1}, S_{w2}	Real and imaginary parts of the dimensionless complex stiffness in vertical direction (w)
$S_{\psi1}, S_{\psi2}$	Real and imaginary parts of the dimensionless complex stiffness in rotation or rocking (ψ)
t	Time
t_m	Thickness of inner soil medium
u	Horizontal displacement of pile
u_1, u_2	Horizontal end displacements of the element
V_{La}	Lysmer's analog velocity
V_n	Shear wave velocity of soil in the lowest layer

V_s	Shear wave velocity of the soil
w	Vertical displacement of pile
w_1, w_2	Vertical end displacements of the element
w_{res}	Resonant amplitude of vertical vibration
W	Weight of eccentric rotating part in oscillator
W_s	Total weight of footing or total static load on pile
x_i, x_j	Distances of each pile from centre of gravity of foundation
y	Pile deflection
y_c	Critical pile deflection
z_c	Height of the center of gravity above the footing base
z_e	Height of horizontal excitation force above the center of gravity of footing

Greek symbols

α	Complex dynamic interaction factor
α_1	Real part of the complex dynamic interaction factor
α_2	Imaginary part of the complex dynamic interaction factor
α_r, β_r	Rayleigh damping coefficients
α_{ij}	Dynamic interaction factor of pile group
α_{st}	Static interaction factor
$[\alpha]$	Matrix of interaction factors
$[\alpha]_H$	Interaction matrix for horizontal translation and rotations
$[\alpha]_V$	Interaction matrix for vertical translation
β'	Frequency-independent soil material damping
β	Angle of pile with direction of loading
δ	Loss angle
δ_r	Relative embedment
δ_{dij}	Dynamic displacement of pile i due to load on pile j
δ_{sij}	Static displacement of pile j due to its own load
$\{\delta\}$	Vector of displacements

List of Symbols

θ	Angle of eccentricity
ϕ	Friction angle of soil
φ	Angle between the direction of load and the plane in which piles lie
ω	Circular frequency
ω_1, ω_2	The frequencies corresponding to the points of interaction between the response curve and a line passing through the origin of coordinates
ω_i	The components of the rotational vector
ω_m	Damped resonant circular frequency
ω_n	Undamped resonant circular frequency
ω_{n1}, ω_{n2}	Two resonant frequencies of coupled vibration
μ	Mass of the pile per unit length
ρ	Mass density
ν	Poisson's ratio of soil
ν_m	Poisson's ratio of inner zone
ν_s	Poisson's ratio of outer zone
λ, λ'	Real part and imaginary part of Lamé's constant
ξ	Effective mass coefficient
ψ	Rotation (rocking) in vertical plane
ψ_1, ψ_2	Rotational (rocking) end displacements of the element
ψ_r	Amplitude of rocking motion
$\psi_{r1-res}, \psi_{r2-res}$	First and second resonant amplitudes for rocking motion
$\varepsilon_{2i,2j}^H$	Complex elements of $[\alpha]_H^{-1}$ that correspond to the moments associated with rotations
$\varepsilon_{2i-1,2j-1}^H$	Complex elements of $[\alpha]_H^{-1}$ that corresponds to the horizontal forces associated with horizontal displacements
ε_{ij}^V	Elements of $[\alpha]_V^{-1}$
Λ	Complex frequency parameter
Δ	Relative volume change
Δ_h	Maximum amplitude of pile for horizontal vibration

Δ_r	Maximum amplitude of pile for rocking vibration
Δ_v	Maximum amplitudes of pile for vertical motion
Ω	Backbone curve

Abbreviations

BEM	Boundary element method
BH	Bore hole
C.G.	Center of gravity
DC	Direct-current
Deg	Degree
DOF	Degree of freedom
Eq.	Equation
FE	Finite element
FEM	Finite element method
Fig.	Figure
GER	Group efficiency ratio
Hz	Hertz
IS	Indian standard
kN	Kilo Newton
L.L.	Liquid limit
m	Meter
mm	Millimeter
M.P.F.	Mass participation factor
P.L.	Plastic limit
Rad	Radian
RPM	Revolutions per minute
SDOF	Single degree of freedom
SPT	Standard penetration test
USCS	Unified soil classification system
2-D	Two dimensional
3-D	Three dimensional