SOME ASPECTS OF NONLINEAR LASER-PLASMA INTERACTION

by

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ABSTRACT

The present thesis studies some of the nonlinear phenomena occurring in EM wave-plasma interactions. The phenomena of self-focusing and the self-trapping of Gaussian beams have been studied in both the unmagnetised plasmas and the magnetoplasmas. Moment method has been employed to study the above-mentioned effect. Studies done with this approach are compared with similar studies done in recent past using the paraxial-ray approximation. For collisional unmagnetised plasmas with saturating nonlinearity, it is found that the moment method predicts the self-trapped beam radii to be independent of the beam power in the saturation region; the paraxial approach however, allows the self-trapped beamwidth to rise rapidly after attaining a minimum value. The divergence between the two theories is more marked for plasmas dominated by electron-ion collision. In plasmas dominated by the electron-neutral particle collisions however, the predictions of the two theories converge as the beam power is gradually raised. In magnetoplasmas, similar studies have been done when circularly polarised Gaussian beams propagate along the static magnetic field. This study too, shows poor agreement between the two theories for both the low frequency ($\omega < \omega_c$) and the high frequency waves ($\omega > \omega_c$). The disagreement is acute

near the gyro-resonance for a right handed circularly polarised wave. Studies have also been done on the beamwidth variation when a Gaussian beam propagates transverse to the dc magnetic field. The effect of self-focusing on the growth rates of filamentation instability in both the collisionless and the collisional plasmas has also been analysed. Besides these, the excitation of upper hybrid wave at the pump frequency by a Gaussian laser beam in extraordinary mode has also been studied and the dependence of power on the self-focusing of the pump has been examined. Further, the second harmonic generation by the interaction of the pump and the excited upper hybrid wave has also been analysed. In addition to the above excitation process, excitation of lower hybrid wave by the beating of two microwaves has also been studied. Finally, rigorous analysis of second harmonic generation by ponderomotive mechanism in a plasma-filled cylindrical waveguide has been made and substantial transfer of power to this harmonic has been predicted.

PREFACE

The first spectacular manifestation of nonlinearity in the interaction of electromagnetic waves with plasmas was the "Luxembourg Effect"[1-4], in which a perturbation in the electron collision frequency in the ionosphere, induced by the propagation of a modulated electromagnetic wave, gets superimposed on a second wave propagating in the same region. However, the present trend of work on nonlinear wave-plasma interaction may be said to be established nearly two decades ago[5-8]; the last decade itself saw prolific publication of literature including monographs and reviews[9-13]. Academic interests apart, the motivation for such studies remains, of course, the necessity of understanding the physics related to various fusion schemes. Besides this, nonlinear interactions find applications in space plasma physics and astrophysics.

Nonlinearity is fundamental to wave-plasma interaction because, the equations of plasma dynamics are themselves nonlinear[14]. For a collisional plasma if the wave amplitude[15],

$$E_0 << \left[3 m_e k_B T \delta \left(\omega^2 + \nu_{eff}^2 \right) / e^2 \right]^{1/2}, \qquad (P1)$$

the nonlinearity may be taken to be weak. (here m_e = electronic mass, k_B is the Boltzmann constant, ω the wave frequency, δ the fractional loss of energy in collision between electron and ion and ν_{eff} is the effective collision frequency). A similar criterion for collisionless plasma is[16]

$$E_0 << \left[e^2/(8 m_e k_B T \omega^2) \right]^{-1/2} \qquad (P2)$$

It is thus apparent from Eqs. (P1) and (P2) that at high frequencies, nonlinearities in plasma are weak. In the absence of nonlinear effects a simple relationship exists between D and E, viz.,

$$D_\mu(\underline{r},t) = \int_{-\infty}^{t} dt_1 \int_{-\infty}^{\infty} d\underline{r_1}\, \epsilon_{\mu\nu}(t-t_1, \underline{r}-\underline{r_1})\, E_\nu(t_1,\underline{r_1}), \qquad (P3)$$

where the arguments of the tensor $\epsilon_{\mu\nu}$ arise because of the temporal and the spatial dispersion. For weak nonlinearities, relation (P3) is replaced by[17]

$$D_\mu(\underline{r},t) = \sum_{n=1}^{\infty} \int_{-\infty}^{t} dt_1 \int d\underline{r_1} \int_{-\infty}^{t_1} dt_2 \int d\underline{r_2} \cdots\cdots \int_{-\infty}^{t_{n-1}} dt_n \int d\underline{r_n}$$

$$\times\, \epsilon_{\mu\nu_1\nu_2\cdots\nu_n}\left(t-t_1, \underline{r}-\underline{r_1}, \underline{r_1}-\underline{r_2}, t_1-t_2, \cdots\right)$$

$$\times\, E_{\nu_1}(t_1,\underline{r_1})\, E_{\nu_2}(t_2,\underline{r_2}) \cdots\cdots E_{\nu_n}(t_n,\underline{r_n}) \qquad (P4)$$

In many situations of interest, retention of only a few number of terms in Eq. (P4) is considered sufficient.

An appreciation of the contribution of the present thesis can best be made by describing it along with a survey of the relevant nonlinear phenomena.

Parametric Excitation

It is the name given to a large class of nonlinear processes recognised as essential to nonlinear mode coupling[12]. It is also one of the most important mechanism of nonlinear mode conversion from electromagnetic to electrostatic and from high frequency to low frequency waves. The process may itself be defined as the amplification of certain oscillation by the periodic modulation of some parameter characterizing the oscillation. As an example, let us consider the propagation of a large amplitude electromagnetic wave (pump) $(\omega_0, \underline{k_0})$ in plasma, where it interacts with some mode $(\omega_1, \underline{k_1})$ of the plasma, resulting in the generation of side-bands $(\omega_0 \pm \omega_1, \underline{k_0} \pm \underline{k_1})$. If one of the side-bands also happen to be a mode $(\omega_2, \underline{k_2})$ of plasma, then the latter will have a net amplification, if the amplification is able to overcome the damping. Similarly, the first mode $(\omega_1, \underline{k_1})$ will also be amplified through the beating of the pump and the second mode $(\omega_2, \underline{k_2})$; thus resulting in the amplification of both the modes. The condition for resonant excitation is thus[9]

$$\omega_0 = \omega_1 + \omega_2 \ , \ \underline{k_0} = \underline{k_1} + \underline{k_2} \ .$$

When both $(\omega_1, \underline{k_1})$ and $(\omega_2, \underline{k_2})$ happen to be electrostatic modes, the process is called decay instability. If however, one of them is an electromagnetic wave, we call the process scattering. In stimulated Raman scattering, the electrostatic wave is a high frequency wave (electron plasma wave), while in the stimulated Brillouin scattering it is a low frequency wave (ion-acoustic wave). In nonlinear Landau damping, the beat of $(\omega_0, \underline{k_0})$ and $(\omega_1, \underline{k_1})$ keeps phase with plasma particles, i.e.,

$$(\omega_0 - \omega_1) = (\underline{k_0} - \underline{k_1}) \cdot \underline{v}$$

where $\underline{v}$ is the drift velocity of the particles. This instability results in the pump decaying into an electromagnetic wave and resonant particles. A thorough study of the growth rates and threshold powers of the various channels of parametric decay was undertaken by Nishikawa[18], Drake et al.[19] Nishikawa[18] had also predicted the existence of a purely growing mode called the "Oscillating Two Stream Instability", when $\omega_0 < \omega_1$. Experimental evidence for this instability has been obtained by Chang and Porkolab[20]. Before that, decay instability was also studied by a few workers[21,22].

In magnetised plasma, unified treatment of the parametric coupling of waves was undertaken by Aliev et al,[23] Amano and Okamoto[24] and Porkolab[25]. Such plasmas are capable of supporting a large number of modes, many of which are strongly damped. This makes the parametric excitation in these plasmas more attractive for heating. Recent discovery of self generated magnetic fields[26-33] of the order of mega-gauss in laser fusion experiments have added a new dimension of interest to this area. On account of these self-generated fields, parametric excitation near the upper hybrid and the lower hybrid frequencies have attained much significance[34-43]. Larson[44] and Johnston and Kaufman[45] obtained a general expression for the coupling coefficients of the three wave decay of electromagnetic waves in a magnetised plasma. Porkolab et al[46] and Grek and Porkolab[47] have observed significant electron heating in presence of decay instability. A review of the comparison between the theory and the experiment on nonlinear effects in plasmas has been given by Porkolab and Chang[48].

Modulation and Filamentation Instability

This instability refers to the perturbation of the envelope of the pump wave as a result of nonlinear interaction between the pump $(\omega_0, \underline{k}_0)$ and a low frequency electrostatic perturbation $(\omega, \underline{k})$. Here, $\omega << \omega_0$ and $k << k_0$ and hence both the side-bands $(\omega_0 \pm \omega, \underline{k}_0 \pm \underline{k})$

become resonant[19]. This instability has low threshold power and has the largest growth rate in the forward direction, i.e., when both the pump and the perturbation propagate in the same direction.

If however, the electrostatic perturbation propagates normal to the pump, a purely growing instability, called the filamentation instability, results. This instability has low threshold power and results in the formation of regions of increased and decreased particle density and hence alternate streaks of increased and decreased light intensity can be observed[49]. Explanation for this instability was given by Bespalov and Talanov[50]. Later on, the analysis of this phenomenon was also made by Kaw et al[51], Perkins and Valeo[52], Drake et al.[19] etc. Bingham and Lashmore-Davies[53], using two fluid model, recovered the growth rates obtained by Drake et al. for the filamentation of a uniform pump wave. Filamentation of whistlers was earlier observed by Stenzel[54] in laboratory plasma, which was theoretically explained on the basis of collisional nonlinearity[55] and antenna effects[56].

In this thesis we have[57] analysed the filamentation instability for a Gaussian pump and obtained substantial enhancement in the growth rates on account of the self-focusing of the pump.

Excitation via Nonlinear Wave Mixing

An alternative way of transferring energy to plasma is by resonantly exciting the natural modes through non-linear wave-mixing (Beating). Here, two waves $(\omega_1, \underline{k_1})$ and $(\omega_2, \underline{k_2})$ interact nonlinearly in a plasma, exciting electrostatic oscillation at $\Delta\omega = \omega_1 \sim \omega_2$ and $\Delta \underline{k} = (\underline{k_1} \sim \underline{k_2})$. If $(\Delta\omega, \Delta\underline{k})$ happen to be a mode of the plasma, resonant excitation takes place. Since $\Delta\underline{k}$ can be suitably adjusted by properly choosing the orientation of $\underline{k_1}$ and $\underline{k_2}$, it is always possible to satisfy the dispersion relation of the mode to be excited. Kroll, Ron and Rostoker[58] proposed wave mixing as density probe for a plasma. The general theory of nonlinear wave mixing in plasma was given by Tsytovich and Shovartsburg[59]. Rosenbluth and Liu[60] studied the field amplitudes of the excited plasma wave in an unmagnetised plasma assuming a linear density profile; the two pump waves propagate in either the same or the opposite directions. But they did not consider the effect of excited wave on the two pumps. A study to this effect was done by Beaudry[61]. Schmidt[62], analysing the resonant excitation of electrostatic mode in an unmagnetised plasma by two oppositely travelling wave, found damping for higher frequency pump and an enhancement of the lower frequency pump. Kaufman and Cohen[63], analysing the excitation of electrostatic wave through wave beating by considering only the local dielectric constant, visualised an efficient plasma heating through successive transfer of energy via a cascade of waves.

In magnetoplasmas, the above study was undertaken by Weyl[64] who analysed the upper hybrid and the plasma wave resonances in a cold, homogeneous plasma. Willet and Maragechi[65] obtained an expression for the power transferred to a magneto-plasma through the beating of two oppositely directed electromagnetic waves and studied the dependence of power on pump intensities and direction of propagation. For pumps having Gaussian distribution of intensity, excitation of natural modes in a plasma is much more interesting and challenging as compared to the situation when uniform pumps are employed. A Gaussian pump, which is used in laser-fusion experiments, while traversing a plasma renders it inhomogeneous by redistributing the carrier density and thereby gets self-focused[16]. This makes the analysis of the problem difficult and cumbersome. Sodha et al.[66-69] studied the excitation of natural modes in plasmas by nonlinear wave mixing of Gaussian pumps and analysed the effect of self-focusing on the excited modes in a number of situations. A study of the excitation of lower hybrid wave through the nonlinear wave beating of two Gaussian pumps is a part of the present thesis.

Besides the above process of mode excitation through wave beating, a single wave can also excite electrostatic oscillations in a plasma provided the pump field has a nonvanishing component along plasma inhomogeneity[70]. In magnetoplasmas, the component of $\underline{V} \times \underline{B}_0$ force along the propagation direction can also drive density fluctuation leading to electrostatic field generation. White and Chen[71] made an exhaustive study of resonance excitation, and amplitude variation of the pump in both, magnetised and unmagnetised plasmas. Hiroe and Ikegami[72] observed lower hybrid oscillation excited by a microwave in extra-ordinary mode. In the present thesis, excitation of an upper hybrid wave by a Gaussian beam has been studied taking into account its self-focusing effects.

Harmonic Generation

Generation of sub-harmonic and higher harmonics has been the subject of interest for quite a long time[73-80]. Sub-harmonic at $\omega/2$ and higher fractional harmonic at $3\omega/2$[81] have been observed in experiments on laser irradiance of targets. Sub-harmonic at $\omega/2$ is generated when a laser beam enters the quarter critical surface in plasma. Generation of $3\omega/2$ frequency was subsequently investigated by Liu and Rosenbluth[82]. Besides the fractional harmonics, integral harmonics have also been investigated[78,79] in infinite plasmas. In a bounded plasma, Kondratenko[73] investigated harmonic generation.

In a collisionless plasma, beat of the pump wave with itself[83] or with the excited plasma wave[84] is responsible for the second harmonic generation. The self beating is provided by the ponderomotive mechanism[85]. In collisional plasmas however, modulation of collision frequency at by the pump intensity through the temperature dependent collision frequency[15], is responsible for third harmonic generation. Sodha and Kaw[86] have given a review of harmonic generation in unbounded collisional plasma. Self-generated magnetic field in laser plasma interaction can also provide an additional mechanism for the second harmonic generation through the Lorentz force $\underline{V} \times \underline{B}$[87]. In the present thesis, second harmonic generation, through ponderomotive mechanism, has been studied in a plasma-filled cylindrical waveguide.

Self-Focusing

It is one of the simplest among the nonlinear phenomena. In this process, a self contraction in the transverse dimension of a beam takes place on account of nonlinearity induced in the medium by the transverse inhomogeneity of the beam. Unlike the parametric instability or harmonic generation, quasi-monochromaticity of the wave undergoing self-focusing is retained. It then follows that in the expansion of $D_\mu(\underline{r},t)$ given by Eq. (P2), we retain only those terms which contain even rank tensors like $\epsilon_{\mu\nu_1\nu_2\nu_3}$, $\epsilon_{\mu\nu_1\nu_2\nu_3\nu_4\nu_5}$ etc. Again these tensors are real if the medium does not exhibit absorption.

The first experimental evidence of self-focusing came from Pilipetskii and Rustamov[88] who photographed thin filaments in organic liquids traversed by a ruby-laser beam. Earlier, the theoretical possibility of this phenomenon in a nonlinear medium was analysed by Askaryan[89], and the profile of a self-trapped beam was calculated by Talanov[90]. However, the most convincing experiment, which initiated extensive theoretical investigation, of the phenomenon came from Garmire, Chiao and Townes[91]. Soon, it was followed by the theory of aberrationless self-focusing by Talanov[92], Kelley[93] and Akhmanov et al.[94]. Aberrational self-focusing was analysed by Dyshko et al.[95] The approach of Akhmanov et al.[94], which is based on the WKB and the paraxial approximations, was later on extensively applied in solids, semiconductors and plasmas[16,96]. In ionosphere, self-focusing was investigated by Litvak[97]. In plasmas, short time scale $(t \ll t_E)$ self-focusing is dominated by the ponderomotive force[85] and the relativistic[98,99] nonlinearity, while the long time scale focusing $(t \gg t_E)$ is dominated by thermal effects[16] (t = pulse time, t_E = energy relaxation time of electrons with heavy particles) Hora[100] has studied the ponderomotive self-focusing of laser beams; thermal self-focusing was studied by Eremin et al.[101] Relativistic nonlinearity in the self-focusing of circularly polarised light was considered by Max[102], Siegrist[103] and Kane and Hora[99]. Sodha and Tripathi[104] investigated self-focusing in plasmas with saturating nonlinearity.

All the above studies have, however, been done by considering only regions close to the axis. Vlasov et al.[105] had, however, formulated an alternative approach to self-focusing phenomenon, which was based on the consideration of certain moments. Sodha, Sinha and Sharma[106], and Sinha and Sodha[107] recently applied this approach for studying the above mentioned phenomenon in plasmas, these publications form a part of the present thesis. Mention must also be made of recent attempts to analyse the problem using the variational approach[108]. This approach too, yields the same critical power for a collisionless plasma as that obtained from the moment theory approach. However, the dynamics of the self-focused beam near the focal point and its structure there are some of the vexing problems of self-focusing which remain unsolved so far.

In the present thesis, some of the nonlinear phenomena mentioned earlier have been studied. A chapterwise summary of the thesis is as follows:-

Chapter-1
Self-Trapping of a Gaussian Beam in Unmagnetised Collisional Plasma

In this chapter the author has studied the self-trapping of a Gaussian laser beam in unmagnetised collisional plasmas by employing the moment method of vlasov et al.[105]. WKB approximation has been used and the plasma is assumed to be aberrationless and absorptionless. Condition for self-trapping and expression for self-trapped beam radius has been obtained and compared with the corresponding quantity obtained from the paraxial ray approximation[16]. It is found that the two theories are in disagreement by a wide margin.

Chapter-2
Self-focusing and Self-trapping of a Gaussian Laser Beam in Magnetoplasma

This chapter is subdivided in sections A and B. Section A deals with the self-focusing of circularly polarised laser light travelling along the direction of applied magnetic field ($\underset{\sim}{B}_0$). Self-focusing and self-trapping of both the right and the left handed circularly polarised beam have been studied from the Moment theory approach under the validity of the WKB approximation. The self-trapped beam radii have been compared with the corresponding figures derived from the paraxial ray approximation for both the collisional and the ponderomotive type nonlinearity.

Similar studies have also been done for the whistlers. It is found that the agreement between the two theories is poor especially near gyroresonance ($\omega_0=\omega_c$) for both the high and the low frequency modes.

Section B deals with the transverse self-focusing of the upper hybrid laser radiation having Gaussian distribution by employing the same technique as for section A for ponderomotive nonlinearity. Variation in the self-trapped beam radius with incident power and also with the static magnetic field has been studied.

Chapter-3
Filamentation of a Gaussian Laser Beam in Plasma

This chapter studies the effect of self-focusing on the growth of filamentation instability driven by the growing nonlinear interaction between the pump wave and sidebands. Two fluid model has been employed for arriving at the dispersion relation under the WKB approximation. It is found that the self-focusing of the pump wave produces significant variations in both the temporal and the spatial growth rates as the laser beam traverses plasma. In the trivial case of uniform pump wave, the growth rates have been shown to reduce to that given by Drake et al.[19]

Chapter-4
Excitation of an Upper Hybrid Wave and Second Harmonic Generation by a Gaussian Beam in Extra-Ordinary Mode

Here we have studied the excitation of an upper hybrid wave and second harmonic generation when a Gaussian beam propagates normal to the static magnetic field in extra-ordinary mode in a magnetoplasma. Expressions for intensity and power of the generated waves have been obtained by taking into account the self-focusing of the pump wave when the pump frequency (ω_0) is far off the resonance, i.e. $\frac{\omega_{uH}}{\omega_0} << 1$ (ω_{uH} being the frequency of the upper hybrid oscillation). It is found that even for the case of non-resonance $\left(\omega_0 >> \omega_{uH}\right)$, significant fraction of incident power ($\sim$ .01%) is transferred to the excited plasma wave. This transfer of energy becomes more and more significant as the pump wave gets progressively self-focused.

Chapter-5
Lower Hybrid Wave Excitation by Beating of Two Microwaves

Excitation of lower hybrid wave has been studied in hot magnetoactive plasma by the beating of two microwave pumps, travelling normal to the static magnetic field ($\underline{B}_0$) in ordinary mode ($\underline{E} \times \underline{B}_0 = 0$). Using two fluid model for plasma with the Maxwell's equations, an equation for the generated electrostatic wave has been set up. Focusing of the excited lower hybrid wave has been predicted when the two pumps have Gaussian distribution of intensity along their wavefronts. Expressions for intensity and power of the generated wave have also been obtained and evaluated numerically. Substantial transfer of power $\sim$ .01% has been predicted in the process of excitation.

Chapter 6
Second Harmonic Generation in a Plasma Filled Cylindrical Wave Guide

This chapter deals with the nonlinear second harmonic generation in a plasma-filled conducting cylindrical wave guide when the electromagnetic wave propagates as TE_{oq} mode. The nonlinearity arises from the time dependent ponderomotive force at frequency 2ω . It is found that the generated harmonic is a mixed wave (partly transverse and partly longitudinal). Expression for power transferred to the generated harmonic has been obtained and evaluated numerically for different plasma densities and also for q = 1 and 2. Dimensional effects on the power transfer has also been studied numerically.

The work reported in the thesis has resulted in the following publications/communications:-

1. The self-focusing of laser beams in magnetoplasma; Moment theory approach, J. of Physics D: Applied Phys. 12, 1079 (1979).
2. Transverse self-focusing of a Gaussian beam: Moment method, Phys.Rev.A (In Press) 1980
3. Filamentation of a Gaussian laser beam in a plasma, Plasma Physics 21, 283 (1979).
4. Excitation of an upper hybrid wave and second harmonic generation by a Gaussian beam in extra-ordinary mode, (Communicated).
5. Lower hybrid wave excitation by beating of two micro-waves, (Communicated).
6. Second harmonic generation in plasma filled cylindrical waveguide, (Communicated).

CONTENTS