NONLINEAR PERIODIC RESPONSE OF CURVED BEAMS AND OVAL CYLINDRICAL SHELLS

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by

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Submitted
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Certificate

This is to certify that the thesis entitled "***Nonlinear periodic response of curved beams and oval cylindrical shells***" being submitted by **Syed Muhammad Ibrahim** to the Indian Institute of Technology Delhi for the award of the degree of Doctor of Philosophy in Applied Mechanics is a record of original bonafide research work carried out by him under our supervision and guidance. The thesis work, in our opinion, has reached the requisite standard fulfilling the requirements for the degree of Doctor of Philosophy.

The results contained in this thesis have not been submitted in part or full to any other University or Institute for the award of any degree or diploma.

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Abstract

Curved beams and circular/noncircular cylindrical shells under dynamic loading may undergo vibration with amplitude of the order of their thicknesses and the associated steady state response involves the participation of multiple modes with sub- and super-harmonics. It is revealed from the literature that most of the studies are based on a set of nonlinear ordinary differential equations derived by using Galerkin method employing spatial discretization with limited mode approximations/expansions which may not capture *a priori* unknown nonlinear modal interactions/phenomena of continuous systems. The effect of excitation in the neighborhood of different symmetric and antisymmetric modes on the nonlinear response of curved beams and the nonlinear dynamics of noncircular cylindrical shells are not dealt with in the literature.

A computationally efficient solver based on Newmark's time marching coupled with shooting technique and arc-length continuation for predicting the nonlinear periodic response of large dynamical systems is successfully employed to predict the nonlinear dynamic response of isotropic/composite curved beams/oval cylindrical shells. Finite element and finite strip methods are used for spatial discretization of curved beams and cylindrical shells, respectively. Several new results and interesting nonlinear phenomena are presented for isotropic/laminated curved beams and cylindrical shells.

The stable/unstable periodic response of isotropic/cross-ply composite curved beams subjected to harmonic excitation with frequency in the neighborhood of $1^{st}/2^{nd}$ symmetric and antisymmetric modes is obtained. 2:1 internal resonance between 2^{nd} antisymmetric and 1^{st} symmetric modes is observed for the first time in the thesis. 1:1 internal resonance exists for a narrow range of the linear frequency ratios of the first two modes.

The effects of ovality parameter and loading locations on the nonlinear steady state periodic response of the oval cylindrical shells excited in the neighborhood of first AS/SA/SS/AA linear free vibration modes are investigated. It is found that for circular cylindrical shells, the contribution of modes with different circumferential wave numbers is responsible for traveling waves emanating at loading point but moving in opposite directions. The secondary branch response due to 1:1 internal resonance between AS/SS and SA/AA modes through the symmetry breaking bifurcation and waves traveling in anticlockwise direction from loading point are presented for the first time for symmetrically excited oval shell. For nonsymmetrically excited oval shells, either waves moving in anticlockwise direction from the loading point or those originating near major axis moving in opposite directions in the top and bottom halves of the shell are observed in certain forcing frequency range.

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