A THESIS ON

WAVES AND FLOWS IN

RADIATION-MAGNETOGASDYNAMICS

By

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Submitted to the Indian Institute of Technology, Delhi, for the award of the degree of Doctor of Philosophy in Mathematics

1970

ACKNOWLEDGEMENTS

I am extremely grateful to Dr. O.P. Bhutani, M.Sc., Ph.D., Department of Mathematics, Indian Institute of Technology, Delhi, for his valuable guidance and encouragement throughout the preparation of this thesis.

I take this opportunity to express my gratitude to Dr. M.K. Jain, M.A., D.Phil., D.Sc., presently Visiting Research Professor, Department of Computer Science, University of Illinois, Urbana, Illinois, U.S.A., for his constant encouragement.

I thank Dr. M.P. Singh, Head of the Department of Mathematics, Indian Institute of Technology, Delhi, for his interest in my work.

I am very grateful to Mr. N.R. Krishnan, Research and Development Organization for Electrical Industry, Piplani, Bhopal, for many helpful discussions.

My thanks are due to the authorities of the Indian Institute of Technology, Delhi, for providing me with research facilities for this work.

Finally I thank Mr. Ranjit Kumar for his patient typing of this thesis.

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CERTIFICATE

This is to certify that the thesis entitled 'WAVES AND FLOWS IN RADIATION-MAGNETOGASDYNAMICS' being submitted by Miss Lakshmi Raman to the Indian Institute of Technology, New Delhi, for the award of the Degree of Doctor of Philosophy in Mathematics, is a record of bonafide research work carried out by her. Miss Lakshmi Raman has worked under my guidance and supervision for two and a half years and has fulfilled the residential and other requirements for the submission of this thesis, which to my knowledge, has reached the requisite standard. The results contained in this thesis have not been submitted, in part or in full, to any other University or Institute for the award of any degree or diploma.

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SYNOPSIS

Except for Chapter I which constitutes an introduction to the entitled thesis and Chapter VI where a general theory for the study of thin airfoils in Radiation-Magnetogasdynamics is developed, the remaining Chapters II-V deal with the macroscopic and non-relativistic study of radiation effects on hydromagnetic waves. The medium is assumed to be inviscid and the non-equilibrium effects due to processes such as dissociation, vibrational relaxation etc. are neglected. Further for the account of the radiation effects the usual quasi-equilibrium hypothesis is assumed. A brief resume of each of the Chapters is given below.

Chapter I: Introduction

Here, after sketching the importance of the interaction of the radiation field with the magnetogasdynamical field we have given the basic equations governing the motion of hot ionized gases and discussed the form of the radiative transfer equation both, for grey and non-grey gases. This is followed by a short historical survey of the related works on flows and wave propagation in Radiation-gasdynamics.

Chapter II: General Three-Dimensional Wave Propagation in Radiation Magnetogasdynamics.

In this Chapter we have carried over the most comprehensive and detailed study of hydromagnetic waves as made by Banos to the field of Radiation Magnetogasdynamics. More specifically, using Banos' technique, we have studied the effects of finite electrical conductivity of the fluid and radiation on general three dimensional wave motion. It is seen that radiation has no effect on the velocity modes (which are devoid of pressure fluctuations and hence are independent of the velocity of sound in the medium). The pressure modes (which are accompanied by pressure fluctuations) are very much modified by radiation. Thus the discussion is confined to the study of pressure modes, for different values of the conductivity and radiation parameters. After obtaining the qualitative picture of the various phenomena occuring through the analytical approach, we have, unlike Banos resorted for a quantitative picture to numerical methods. In fact tables have been provided to show how the difgerent modes behave. Both plane and cylindrical waves have been discussed. In general the effect of radiation is to attenuate slightly the modified sound wave and the modified Alfven wave. Also on entirely new mode termed as the 'radiation induced wave' is introduced. Further in the case of cylindrical waves, it is shown that the radiation induced modified sound wave and the radiation induced modified Alfven wave can propagate for all values of the transverse wave number. The last result has no counter part in conventional magnetogasdynamics.

Chapter III: On Plane Linear Radiation-Magnetogasdynamic Waves.

This Chapter utilizes the apparatus of plane asymptotic wave solution combinations to make a systematic study of the effects of radiation and finite electrical conductivity on plasma flows. More specifically we have discussed the perturbation modes of the steady parallel flow of an inviscid, finite electrically conducting and radiating gas in a uniformly oriented magnetic field. The complete study is divided into two sections. Section I deals with radiating plasma flows with infinite conductivity and Section 2 with radiating plasma flows with finite electrical conductivity. Using asymptotic technique as developed by Lagerstrom, Cole and Trilling, we have shown that the transverse mode is not affected by radiation, while the effect of radiation on the longitudinal mode is to impose a general diffusion pattern with the diffusion coefficient depending on the Mach number and the external magnetic field. This is to be contrasted with the role played by viscosity where the diffusion coefficient depends only on the magnetic Reynold's number. Further for obtaining the qualitative and quantitative picture of the phenomena occuring, we have drawn the velocity phase diagram. This diagram when seen under the limitations involved, becomes a very good aid for visualising the effects of various parameters.

Chapter IV: An Approximate Method for Analysing Harmonic Disturbances in Radiation-Magnetogasdynamics.

The main aim of this chapter has been to establish the equivalence of the single fifth order equation obtained for the divergence of the velocity vector with a set of lower order equations. For this purpose the problems of propagation of small disturbances generated by the harmonic oscillation of a planar wall is considered. The exact solutions to the equation for the divergence of the velocity vector and the approximate equations obtained by using extended form of Whitham's technique on the exact equation, are obtained and a number of graphs depicting the behaviour of the amplitude and the damping of waves have been drawn. On account of the magnetic field we have a set of three waves (which in the purely radiating case reduces to two), a radiation-modified slow wave, a radiation modified fast wave and a radiation induced wave. It is seen that the approximatie and exact results agree quite well except over certain regions of the Boltzmann number. These exceptional regions, where the solution switches over from one curve to another, are termed as 'transition regions'. To be more specific, it is shown that the exact equation can be replaced by a set of six lower order equations, and two of these equations taken at a time constitute a wave. Over some regions of the Boltzmann number one of these equations gives a good approximation while over some other regions, the solution of the remaining equation tallies well with the exact equation.

Chapter V: Radiation Waves in An Anisotropic Conducting Medium.

In this chapter we have studied the effect on wave propagation of anisotropic conductivity by reducing it to Hall effect. The complete study is divided into two parts. Part-I deals with one dimensional flow (one independent variable and three components of velocity) while, in part II, the problem of plane steady flows is taken up. Using the usual technique of wave propagation, we have shown that, for one dimensional flow, the velocity can be monotone as well as periodic. Further, for the plane steady flow of a collisionless plasma, the radiation interaction is shown to introduce damping and change the inclination of waves.

Chapter VI: General Theory of Thin Airfoils In Radiation Magnetogasdynamics.

The entitled theory has been developed for the aligned magnetic field case. In fact the problem has been reduced to the solution of a single fifth order partial differential equation in the velocity potential ϕ. Using Green's function technique we have shown that the property of the material body does not modify the flow. A detailed study of the various possible cases such as Supersonic and Super-Alfvenic, Subsonic and Sub-Alfvenic, Supersonic and Sub-Alfvenic, Subsonic and Super-Alfvenic and a remark about the source function are made.

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