

**PROBABILISTIC ANALYSIS OF COMPOSITE PLATES
UNDER LOW AND HIGH VELOCITY IMPACT**

SHIVDAYAL PATEL



**DEPARTMENT OF APPLIED MECHANICS
INDIAN INSTITUTE OF TECHNOLOGY DELHI**

August 2016

©Indian Institute of Technology Delhi (IITD), New Delhi, 2016

**PROBABILISTIC ANALYSIS OF COMPOSITE PLATES UNDER LOW
AND HIGH VELOCITY IMPACT**

By

SHIVDAYAL PATEL

DEPARTMENT OF APPLIED MECHANICS

Submitted

In fulfillment of the requirements of the degree of

DOCTOR OF PHILOSOPHY



INDIAN INSTITUTE OF TECHNOLOGY DELHI

AUGUST 2016

BE GOOD, DO GOOD, BE ONE

*This Ph.D. work is dedicated to My Parents and
Teachers*

CERTIFICATE

This is to certify that the thesis entitled “**Probabilistic Analysis of Composite Plates under Low and High Velocity Impact**” being submitted by **Mr. Shivdayal Patel** is the report of bonafide research work carried by him under our supervision. This thesis has been prepared in conformity with the rules and regulations of **INDIAN INSTITUTE OF TECHNOLOGY DELHI**. We further certify that the thesis has attained a standard required for the award of a **Doctor of Philosophy** degree of the Institute. The research reported and the results presented in the thesis have not been submitted, in part or full to any other institute or university for the award of any other degree or diploma.

Dr. Suhail Ahmad

Professor

Department of Applied Mechanics

Indian Institute of Technology Delhi

Huaz Khas, New Delhi-110016, INDIA

Dr. Puneet Mahajan

Professor

Department of Applied Mechanics

Indian Institute of Technology Delhi

Huaz Khas, New Delhi-110016, INDIA

Dated: August 2016

Place: New Delhi

ACKNOWLEDGEMENTS

I would like to express my heartiest gratitude to my supervisors, Prof. Suhail Ahmad and Prof. Puneet Mahajan, Department of Applied Mechanics, IIT-Delhi, for their valuable guidance that helped me to develop the research skills, and especially for training me to be a better professional in thinking, executing, writing, and presenting my work. I am indebted to their excellent guidance, motivation and support during the entire course of this program. I am also thankful to the faculty and staff of IIT-Delhi for their help and support.

I thank Prof. Santosh Kapuria, Dr. Maloy Kumar Singha and Prof. Naresh Bhatnagar, the members of my Research Committee for their valuable suggestions during my oral and comprehensive examination. Their guidance and suggestion were very useful from giving direction to my research work. I am also thankful Prof. B. P. Patel and all the faculty members of the department who supported and helped me in accomplishing the present research work at IIT Delhi.

I am also thankful to Dr. N. D. Mittal, (Professor and Head MANIT Bhopal), Dr. M S Hora (Asso. Dean MANIT Bhopal), Dr. Kartekeya Tripathi and Prof. Satish B. Purohit (Head SGSITS Indore), who encouraged me to do Ph.D. from IIT Delhi. Their motivation and support at different occasions were valuable to me. I am also thankful to my lab mates and friends Dr. Sangeeta Khare, Dr. Poonam, Dr. Rajneesh Sharma, Dr. Ankur Gupta, Dr. Yaqoob Yasin, Dr. Nilamber Kumar Singh, Dr. Nitin Sharma, Dr. Rabijit, Yudhast Singh, Jitendra, Kartekeya, Arshad, Chandrahas, Rajesh, Vivek, Abhimanyu, Anubhav, Shishodhiya, Ashwin, Ajay Kumar, Vishwanath, Anurag, Mohit, Emrati, Shreya, Gaurav, Rishav, Amit, Babu, Prakhar and Gargi for their valuable time and motivation during this period. I am appreciative to Dr. Suhail Ahmad, Dr. B. P. Patel and Dr. S. Hegde for his great support, motivation and guidance at different stages and aspects of life. Their friendship made me rich and presence makes me happy.

I am also grateful to my beloved wife Rekha Patel and her parents for their patience, motivation and support.

I would like to express my deep sense of gratitude to My Grand Father Sh. Chhote Lal Patel, Grandmother Smt. Ram Bai Patel, Father Sh. Premnarayan Patel, Mother Smt. Girja Devi, and Brother Mr. Shivilakhan Patel for their motivation, support and blessings. I am also highly

indebted to my spiritual Guru Sidhanath Maharaj (Ghaupatala Nand) for his blessings me for achieving this target.

My apologies to my son, Shiva Sheesh Patel for not giving quality life and time to during my Ph.D. work.

Shivdayal Patel

ABSTRACT

The low and high velocity impact response of composite targets is an important investigation to assess its reliability for applications to light weight body and vehicle armors. The work has extensive military, aerospace and civil applications. Safety and reliability assessment as per the international standards is one of the basic objectives of the study. The low and high velocity impacts can cause superficial damages such as surfaces scratches which only significantly affect the structural load bearing capability and may be neglected in the analyses. The damage initiates as an intra ply matrix crack due to shear or bending which propagates further into the interface causing de-lamination between dissimilar plies and fiber breakage. This damage evolves with time and adversely affects the mechanical properties and strength of the composite. Since, multiple cracks in the ply are difficult to track, a progressive damage mechanics approach is used to model this failure. The inter ply failure is modeled using cohesive surfaces between the plies. The progressive damage is modeled in terms of damage energy level. When the damage variable reaches a critical value and stiffness is very low the material is not able to sustain any load further and the element does not participate in further calculations. In order to determine the response of the composite under impact, dynamic analysis of composite plate subjected to low and high velocity impacts has been carried out. Damage initiation and propagation failure due to matrix cracking and delamination are investigated for safety criteria for the low velocity impact. Fiber damage initiation and propagation are considered to attain the safety criteria for the S2-glass epoxy composite beam under high velocity impact. The progressive damage model is adopted and implemented in the FE code as a user-defined subroutine (VUMAT). At the onset of study numerical results are obtained using the deterministic progressive damage model and validated against the existing experimental study in the literature. A linear relation exists between impactor velocities (120 m/s to 300 m/s) and numbers of damaged layers for the symmetric cross ply composite beams. This behavior validates the current finding with respect to the published results.

Probabilistic response assessment is strongly recommended by various international codes, especially to achieve challenging design targets for satisfactory service and survival. It is because, the deterministic approach does not account for the natural scatter in the input data. The probabilistic approach explicitly quantifies the extent of safety or probability of failure.

Many a times, design standards recommend a threshold of target reliability for satisfactory design. Probabilistic approach is, all the more, significant to assess the safety of composite armors. The material properties of composites inherently exhibit a scatter or uncertainty due to their inhomogeneity, anisotropic characteristics, brittleness of the matrix and fiber and manufacturing defects. In the conventional approach, deterministic input data is assumed to participate and associated uncertainties are accounted for by using the deterministic factor of safety. This factor of safety too is uncertain to a greater extent. Hence, there is a persistent need to quantify its performance in a realistic fashion, taking in to account the actual mean and statistics data. At present a probabilistic study is carried out to consider the uncertainties of material properties (elastic modulus, Poisson's ratio, shear modulus, strength properties and fracture energy) and initial velocity. In order to take this stochastic variation into consideration the dynamic analysis is performed using the stochastic finite element method. The required discretization of material properties of random nature is done using Latin hyper cube sampling method, which is employed here due to its relative merits. The stochastic progressive damage model is developed and implemented in the SFE code NESSUS as a user-defined subroutine (VUMAT). The randomly generated sets of properties are used in the stochastic FE analysis to provide the statistics of critical stresses at strategic locations. These stresses and random strengths properties are used for the comparative failure assessment using different failure criteria. On substitution of these critical stresses and respective random strength values, in terms of their statistical characteristics and distributions in the limit state function, a joint probability of failure (P_f) of each ply is obtained. This is numerically evaluated using the Gaussian process response surface method (GPRSM). Multi dimensional integral for determining P_f is calculated by using Gaussian process response surface method (GPRSM). Monte Carlo simulation (MCS) for evaluating the same integral required 10,000 cycles to reach a converged value of P_f and takes almost 40 times more computational time than GPRSM for the same accuracy. This procedure is adopted to investigate the P_f for low velocity impact (LVI) and high velocity impact (HVI) based on damage initiation and propagation models. Comparative study of the probability of failure is carried out using different fiber damage initiation and propagation criteria. The optimum ply arrangement is studied for fiber failure initiation and propagation of composite beams for simply supported and clamped-clamped boundary conditions. System P_f and

sensitivity analysis are also investigated for a composite plate under low and high velocity impact.

The target reliability may be achieved for an optimal combination of the impactor mass and velocity. There is a possibility of underestimation of the peak contact force and displacement by 10.7% and 11.03%, respectively, if the scatter in the material properties and impactor velocity are not considered and mean values are used for calculating response. A 3D stochastic finite element analysis based design optimization of laminated composite beams under high velocity impact is performed to determine the lay-up arrangement having a lower probability of failure (Pf). Comparative study has been carried out, in a novel way, in terms of probability of failure for different ply arrangements, boundary conditions using various failure criteria. Comparative study of different fiber failure initiation criteria is studied and found that a maximum stress criterion is more conservative than the Yen and Hashin failure criterion. There is a possibility of underestimation of maximum areas of the fiber damage and matrix damage by 62.5% and 38.05%, respectively, if the scatter in the material properties and impactor velocity are not considered and mean values are used for calculating response. For the parallel and mixed systems Pf due to fiber damage propagation for clamped-clamped and simply supported BCS of anti-symmetric cross ply arrangement (Case-II) is lesser than that for other ply arrangements, namely, symmetric cross ply (Case-I), symmetric angle ply (Case-III) and anti-symmetric angle ply (Case-IV). Sensitivity based design optimization is carried out for all ply arrangements and different boundary conditions. This is an important input for the stochastic design optimization. The study provides a realistic procedure for design optimization that duly considers the probabilistic scatter in all the properties involved in the analysis and design of composite body armors.

CONTENTS

CERTIFICATE	i
ACKNOWLEDGEMENTS	ii
ABSTRACT	iv
CONTENTS	vii
LIST OF FIGURES	xi
LIST OF TABLES	xiv
NOMENCLATURE	xv
CHAPTER – 1	1
INTRODUCTION.....	1
1.1 Composite Armors	1
1.2 Types of Impact on Composites.....	1
1.3 Modes of Failure	2
1.4 Damage Configuration.....	3
1.5 Risk Assessment of Composite Armors.....	4
1.6 Uncertainties in Composite Properties.....	4
1.7 Probabilistic Assessment of Composites	5
1.8 Stochastic Finite Element Method (SFEM).....	5
1.9 Random Field Discretization	6
1.10 Reliability of Composites.....	6
1.11 Limit State Functions	7
1.12 Reliability Assessment Methods.....	7
1.13 System Probability of Failure	9
1.14 Sensitivity Analysis	10
CHAPTER -2.....	11
LITERATURE REVIEW	11
2.1 Low Velocity Impact (LVI)	11
2.2 High Velocity Impact (HVI).....	14
2.3 Uncertainties Associated with Composite	18
2.4. Stochastic/Probabilistic Finite Element Method (PFEM).....	19
2.5 Random Field Discretization	19
2.6 Reliability of Composites	20

2.7 Existing Gaps in the Literature	25
2.8 Objectives	25
2.9 Organization of the Thesis	26
CHAPTER -3.....	28
DAMAGE INITIATION AND PROPAGATION MODELING FOR LOW AND HIGH VELOCITY IMPACT	28
3.0 Introduction.....	28
3.1 Finite Element Formulation	28
3.2 Constitutive Law	30
3.3 Constitutive Damage Model	30
3.4 Damage Modeling under Low Velocity Impact.....	33
3.4.1 Damage Initiation Model	33
3.4.2 Damage Evolution Model:	35
3.5 Damage Modeling of Composite for High Velocity Impact.....	38
3.5.1 Damage Initiation Criteria	39
3.5.2 Damage Propagation Criteria.....	41
3.6 Modeling of Delamination using Cohesive Behavior.....	43
3.6.1 Modeling of Interface Damage	44
3.6.2 Modeling of Interface Damage Initiation.....	44
3.6.3 Modeling of Interface Damage Evolution.....	45
3.7 Deformable Impactor Specifications	46
3.8 Numerical Study	47
3.8.1 Quasi Static Impact	47
3.8.2 Low Velocity Impact	55
3.8.3 High Velocity Impact.....	60
3.9 Conclusions.....	65
CHAPTER -4.....	66
STOCHASTIC PROGRESSIVE FINITE ELEMENT ANALYSIS OF COMPOSITE UNDER IMPACT	66
4.0 Introduction.....	66
4.1 Uncertainties in FRP Composite.....	66
4.1.1 Discretization of Gaussian Stochastic Processes / Fields	67
4.1.2 Discretization of Non-Gaussian Vector Processes / Fields.....	67

4.2 Probabilistic Design Methodology of Composite Body Armor.....	67
4.3 Stochastic Finite Element Method (SFEM).....	70
4.3.1 Random Field Discretization	71
4.3.2 Stochastic Progressive Damage Modeling.....	74
4.3.3 Probabilistic Response	77
4.4 Limit State Functions.....	78
4.5 Joint Probability Distribution and Failure Region	82
4.6 Reliability Assessment Method	82
4.6.1 Mean Value First-Order Second-Moment Method (MVFOSM).....	83
4.6.2 Second Order Reliability Method (SORM)	83
4.6.3 Monte Carlo Simulation Method (MCSM).....	84
4.6.4 Response Surface Method (RSM).....	85
4.6.5 Gaussian Process Response Surface Method (GPRSM).....	85
4.7 System Probability of Failure	87
4.7.1 Failure of Composite as System in Parallel	88
4.7.2 Failure of Composite as System in Series:.....	89
4.7.3 Failure of Composite as Mixed Systems:	89
4.8 Fault Tree Analysis	90
4.9 Sensitivity Analysis:	90
CHAPTER -5.....	91
PROBABILISTIC STUDY OF COMPOSITE TARGET UNDER LVI &HVI	91
5.0 Introduction.....	91
5.1 Stochastic Finite Element Analysis of Low Velocity Impact	92
5.1.1 Computational Efficiency of Reliability Methods	93
5.1.2 Probabilistic Behavior of Contact Force	94
5.1.3 Probabilistic Behavior of Displacement Time History	96
5.2 Failure Assessment of Composite Plate for Damage Initiation and Propagation	98
5.2.1 Effect of Impactor Mass on Pf for Damage Initiation Model	99
5.2.2 Effect of Impactor Velocity on Pf for Damage Initiation Model.....	101
5.2.3 Effect of Impactor Mass on Pf for Damage Propagation Model.....	103
5.2.4 Effect of Impactor Velocity on Pf for Damage Propagation Model	105
5.3 Stochastic Finite Element Analysis of High Velocity Impact	108

5.3.1 Damage Response With and Without Probabilistic Scatter	109
5.3.2 Computational Efficiency of Reliability Methods in Terms of Sampling Points	111
5.4 Failure Assessment of Composite Plate.....	113
5.5 Comparative Study of Pf for Different Damage Initiation Model (DIM) (Case-A)	113
5.6 Comparative Study of Pf for Different Damage Propagation Model (DPM) (Case-B).....	117
5.7 Conclusions.....	122
5.7.1 Probabilistic Response of LVI	122
5.7.2 Probabilistic Response of HVI.....	123
CHAPTER-6.....	124
SENSITIVITY BASED PROBABILISTIC DESIGN OPTIMIZATION.....	124
6.0 Introduction.....	124
6.1 System Probability of Failure	124
6.1.1 System Pf for Low Velocity Impact	125
6.1.2 System Pf for Damage Propagation Model under High Velocity Impact.....	135
6.2 Sensitivity Analysis	139
6.2.1 Sensitivity Analysis for Low Velocity Impact.....	139
6.2.2 Sensitivity Analysis for High Velocity Impact	147
6.3 Conclusions.....	159
6.3.1 System Probability of Failure and Sensitivity Analysis of Composites under LVI.....	159
6.3.2 System Probability of Failure and Sensitivity Analysis of composites under HVI	160
CHAPTER -7.....	162
CONCLUSIONS	162
SCOPE OF FUTURE WORK	165
REFERENCES	166
LIST OF PUBLICATIONS	176
BRIEF BIO DATA OF THE AUTHOR.....	179

LIST OF FIGURES

Fig.1.1 Modes of Failure.....	2
Fig. 2.1 Mathematical model to study LVI.....	11
Fig. 2.2 Physical system and probabilistic model.....	19
Fig. 3.1 Uniaxial stress-strain relation of damaged material.	31
Fig. 3.2 State of stress on fracture plane during transverse compressive failure.	35
Fig. 3.3 Stress strain behavior in longitudinal direction.	36
Fig. 3.4 Stress strain tensile behavior in transverse direction.	37
Fig. 3.5 Stress strain compressive behavior in transverse direction.	38
Fig. 3.6 Traction separation behavior for interface damage	44
Fig. 3.7 FE mesh for composite plate and Impactor (d=diameter of impactor).....	48
Fig. 3.8 Validation of contact force history	49
Fig. 3.9 Energy time history for an impact velocity of 3 m/s and mass 2.6kg.....	50
Fig. 3.10 Artificial strain energy when mass and velocity of impactor are varied keeping its total energy constant at 11.8 J.....	52
Fig. 3.11 Stress time history in longitudinal direction at impactor velocity of 54m/s.	53
Fig. 3.12 In-plane shear stress (S12) time history at impactor velocity of 54m/s.....	53
Fig. 3.13 In-plane shear stress (S13) time history for an impactor velocity of 54m/s.	54
Fig. 3.14 Stress time histories for an impactor velocity of 3 m/s.....	54
Fig. 3.15 FE mesh for composite plate (25600 elements) and impactor (d=diameter of impactor).	56
Fig. 3.16 Effect of impactor velocity on contact force and validation.....	56
Fig. 3.17 Effect of impactor velocity on time variation of mid-point displacement.....	58
Fig. 3.18 Effect of impactor mass on time variation of contact force (10m/s).	58
Fig. 3.19 Effect of impactor mass on time variation of mid-point displacement of the plate.....	59
Fig. 3.20 Energy balance of symmetric cross ply composite plate.	60
Fig. 3.21 Schematic diagram of composite beam and impactor	61
Fig. 3.22 FE mesh for impactor and composite beam.....	61
Fig. 3.23 Comparison of number of damaged laminae at various velocities predicted from FE simulations with numerical and experimental results of Sevkati et al.....	63
Fig. 3.24 Numerically predicted progressive damage behavior.....	64
Fig. 3.25 (a) Partial Perforation (320 m/s) (b) Full Perforation (400 m/s).....	64
Fig. 4.1 Probabilistic design methodology of composite targets.	69
Fig. 4.2 Comparison of (a) Monte Carlo and (b) Latin Hypercube sampling (LHS).....	73

Fig. 4.3 Parallel system for probability of failure.	89
Fig. 4.4 Series system for probability of failure.	89
Fig. 4.5 Mixed systems for probability of failure.	90
Fig. 5.1 Convergence study of MCS.	93
Fig. 5.2 Convergence study of GPRSM.	94
Fig. 5.3 Probabilistic response of contact force	95
Fig. 5.4 Probabilistic response of contact force with scatter in impactor velocity	95
Fig. 5.5 Probabilistic response of contact force with scatter in material properties	96
Fig. 5.6 Probabilistic response of displacement with scatter in material properties and impactor velocity.	97
Fig. 5.7 Probabilistic response of displacement with scatter in impactor velocity	97
Fig. 5.8 Probabilistic response of displacement with scatter in material properties	98
Fig. 5.9 Pf due to matrix cracking for an impactor velocity of 10 m/s.	99
Fig. 5.10 Pf due to delamination initiation for an impactor velocity of 10 m/s.	101
Fig. 5.11 Pf due to matrix cracking initiation for an impactor mass of 8.66 g.	102
Fig. 5.12 Pf due to delamination initiation for an impactor mass of 8.66 g.	102
Fig. 5.13 Pf due to matrix cracking propagation for an impactor velocity of 10 m/s	103
Fig. 5.14 Pf due to delamination propagation for an impactor velocity of 10 m/s	104
Fig. 5.15 Pf due to matrix cracking propagation for an impactor mass of 8.66 g.	106
Fig. 5.16 Pf due to delamination propagation for an impactor mass of 8.66 g.	107
Fig. 5.17 Effect of scatters of material properties and impactor velocity on fiber damage patterns.	109
Fig. 5.18 Effect of scatters of material properties and impactor velocity on matrix damage patterns.	110
Fig. 5.19 Effect of scatter of material properties and impactor velocity on delamination damage pattern.	110
Fig. 5.20 Comparisons of fiber damage patterns considering the scatter of material properties, which keeping impactor velocity constant at 320 m/s.	111
Fig. 5.21 Pf versus no. of sampling points for MCS method.	112
Fig. 5.22 Pf versus no. of sampling points for GPRSM method.	112
Fig. 5.23 Comparison of Pf of fixed ended composite beams for different ply arrangements and criteria.	116
Fig. 5.24 Comparison of Pf of simply supported composite beams for different ply arrangements and criteria.	117
Fig. 5.25 Comparison of Pf of fixed ended composite beams for different ply arrangements and criteria.	119

Fig. 5.26 Comparison of Pf of simply supported composite beams for different ply arrangements and criteria.	121
Fig. 6.1 Fault tree in series for matrix cracking initiation.....	127
Fig. 6.2 Fault tree in series for delamination initiation.....	127
Fig. 6.3 Fault tree block diagram for matrix cracking initiation of parallel system.....	129
Fig. 6. 4 Fault tree in parallel for delamination initiation.....	130
Fig. 6.5 Fault tree in mixed for matrix cracking initiation.....	130
Fig. 6.6 Fault tree in mixed for delamination initiation.....	131
Fig. 6.7 Fault tree in series for fiber propagation.....	136
Fig. 6.8 Fault tree in parallel for fiber propagation.....	137
Fig. 6.9 Fault tree in mixed for fiber propagation.....	138
Fig. 6.10 Sensitivity of Pf for matrix cracking propagation.....	141
Fig. 6.11 Cumulative distribution function (CDF) of matrix cracking propagation.....	141
Fig. 6.12 Sensitivity of Pf for matrix cracking propagation at impactor velocity of 30 m/s.....	142
Fig. 6.13 CDF for matrix cracking propagation at impactor velocity of 30 m/s.....	143
Fig. 6.14 Sensitivity of Pf for delamination propagation at impactor velocity of 20m/s.....	144
Fig. 6.15 CDF for delamination propagation at impactor velocity of 20m/s.....	145
Fig. 6.16 Sensitivity of Pf for matrix cracking and delamination propagation.....	146
Fig. 6.17 CDF of matrix cracking and delamination propagation at impactor mass of 3M.....	146
Fig. 6.18 Sensitivity of Pf for clamped-clamped composite beam.....	148
Fig. 6.19 CDF for clamped-clamped cross ply composite beam.....	149
Fig. 6.20 Sensitivity of Pf for simply supported composite beam.....	150
Fig. 6.21 CDF for simply supported cross ply composite beam.....	151
Fig. 6.22 Sensitivity of Pf for composite beam at impactor velocity of 300 m/s.....	153
Fig. 6.23 CDF for clamped-clamped cross ply composite beam.....	153
Fig. 6.24 Sensitivity of Pf for composite beam at impactor velocity of 300 m/s.....	154
Fig. 6.25 CDF for clamped-clamped symmetric angle ply composite beam.....	155
Fig. 6.26 Sensitivity of Pf for composite beam at impactor velocity of 300 m/s.....	156
Fig. 6.27 CDF for symmetric cross ply composite beam.....	157
Fig. 6.28 Sensitivity of Pf for composite beam at impactor velocity of 300 m/s.....	157
Fig. 6.29 CDF for symmetric angle ply composite beam.....	158

LIST OF TABLES

Table 3.1: Mode of failure vs. Damage variables	43
Table 3.2 Material and structural properties for glass epoxy composite	49
Table 3.3 Material and structural properties for graphite epoxy composite	55
Table 3.4 Material and structural properties for S2 glass epoxy composite	61
Table 3.5 Numerical data for Johnson-Cook Model.....	62
Table 5.1 Statistical characteristics of material properties and design variables (Graphite epoxy).....	92
Table 5.2 Pf due to matrix cracking initiation for different impactor masses and velocities.....	100
Table 5.3 Pf due to delamination initiation for different impactor masses and velocities.	100
Table 5.4 Matrix cracking propagation for different impactor masses and velocities.	104
Table 5.5 Delamination propagation for different impactor masses and velocities.	105
Table 5.6 Statistical characteristic of material properties and design variables (S2 glass epoxy)	108
Table 5.7 Comparative study of Pf for different ply lay-ups and boundary conditions for damage initiation model (Case-A).....	114
Table 5.8 Comparative study of Pf for different ply lay-ups and boundary conditions for damage propagation model (Case-B)	118
Table 6.1 System Pf due to Matrix cracking initiation for different impactor masses and velocities.....	128
Table 6.2 System Pf due to delamination initiation for different impactor mass and velocity.	128
Table 6.3 System Pf due to matrix cracking propagation for different impactor mass and velocity.	132
Table 6.4 System Pf due to delamination propagation for different impactor mass and velocity.	132
Table 6.5 System Pf due to propagation for Matrix cracking and delamination for different impactor mass and velocity.	134
Table 6.6 Comparison of system Pf due to damage propagation for different ply lay-ups and boundary conditions.....	135

NOMENCLATURE

σ_{ij} , [σ]	Stress at tensor	Y_t	Transverse tensile strength
ρ	Density	Y_c	Transverse compressive strength
\ddot{u}_i, \ddot{u}_B	Acceleration components	σ'_{33}	Normal Transverse tensile stress
u_i, u_B	Displacement components	σ^c_{33}	Normal Transverse compressive stress
b_i	Body force components	Z_t	Normal Transverse tensile strength
ε_{ij} , [ε]	Strain at tensor	Z_c	Normal Transverse compressive strength
$F_{ci}, F_c(t)$	Contact force components	τ_{12}, τ_{13}	In-plane shear stresses
m_i	Mass of impactor	τ_{13}	Out of plane shear stresses
M	Mass of plate	T_{12}, T_{13}	In-plane shear strengths
K, C_0	Stiffness of the plate	T_{23}	Out of plane shear strength
Ω	Effective damage tensor	σ'_n	Normal stress component
d_f^t	Tensile fiber damage	σ'_t	Tangential stress component
d_f^c	Compressive fiber damage	σ'_n	Stress component normal to tangential direction
d_m^t	Tensile matrix damage	σ'_n	Stress component normal to longitudinal direction
d_m^c	Compressive matrix damage	T_{23}^A	Transverse shear strength
d_f	Fiber damage	μ'_n	Frictional coefficient normal and tangential direction
d_m	Matrix damage	μ'_n	Frictional coefficient normal and longitudinal direction
$\dot{\sigma}$	Incremental stress tensor	$\varepsilon'_{f,1}$	Final fiber failure in tension
$\dot{\varepsilon}$	Incremental strain tensor	$\varepsilon'_{0,1}$	Initial fiber failure in tension
$\dot{\Omega}$	Incremental damage tensor	ε_{11}	Longitudinal strain
E_{ij}	Young's Modulii	$\dot{\varepsilon}_{11}$	Incremental longitudinal strain
G_{ij}	Shear Modulii	$\varepsilon^c_{f,1}$	Final fiber failure in compression
ν_{ij}	Poisson's ratio	$\varepsilon^c_{0,1}$	Initial fiber failure in compression

C	Degraded stiffness matrix	ε_{22}	Transverse strain
\dot{d}_f	Incremental fiber damage	$\dot{\varepsilon}_{22}$	Incremental transverse strain
\dot{d}_m	Incremental matrix damage	$\varepsilon_{m,2}^t$	Final matrix failure in tension
D	Damage tensor	$\varepsilon_{0,2}^t$	Initial matrix failure in tension
ε	Strain at any point 't'	$\varepsilon_{m,2}^c$	Final matrix failure in compression
ε_0	Initiation strain	$\varepsilon_{0,2}^c$	Initial matrix failure in compression
ε_f	Final failure strain	λ_r'	Resultant shear strain
Gc	Fracture energy	$\dot{\lambda}_r'$	Incremental resultant strain
σ^0	Initial strength	$\lambda_{12}, \lambda_{13}$	In plane shear strains
L	Characteristics Length	λ_{23}	Out of plane shear strain
T_{ij}	Shear strength	r_i	Damage thresholds
σ_{11}^t	Longitudinal tensile stress	< >	Maclory Bracket
σ_{11}^c	Longitudinal compressive stress	ε_{33}	Out of plane strain
X_t	Longitudinal tensile strength	T_{33}	Out of plane strength
X_c	Longitudinal compressive strength	M, m	Material constant
σ_{22}^t	Transverse tensile stress	ϖ_j	Damage variables
σ_{22}^c	Transverse compressive stress	$\dot{\varpi}_j$	Incremental damage variables
q_{ij}	Damage coupling coefficients	ϖ_1	Fiber damage in tension
δ_n^0	Initial normal separation	ϖ_3	Fiber damage in compression
δ_t^0	Initial tangential separation	ϖ_2	Matrix damage in tension
δ_s^0	Initial shear separation	ϖ_4	Matrix damage in compression
δ_n^f	Final normal separation	ϖ_5	Fiber crushing damage
δ_t^f	Final tangential separation	ϖ_6	Matrix damage in shear
δ_s^f	Final shear separation	t_n	Normal traction
G_n, G_{IC}	Normal fracture energy	t_s	Shear traction
G_s, G_{IIC}	Shear fracture energy	t_t	Tangential traction
G_t, G_{IIIc}	Tangential fracture energy	σ_n	Normal strength
G_n^c	Critical Normal fracture energy	σ_t	Tangential strength

G_s^c	Critical Shear fracture energy	σ_s	Shear strength
G_t^c	Critical Tangential fracture energy	C	Transition temperature
A, B	Material Hardening constants	μ	Frictional coefficient
$\dot{\epsilon}_0$	Strain rate	G_f^t	Fiber fracture energy in tension
θ_m	Melting temperature	G_f^c	Fiber fracture energy in compression
N	Normal strength	G_m^t	Matrix fracture energy in tension
S	Shear strength	G_m^c	Matrix fracture energy in compression
T	Tangential strength	K_{nn}	Normal stiffness
1M	8.66 g of impactor mass	K_{ss}	Shear stiffness
X(t)	Random fields	K_{tt}	Tangential stiffness
$\alpha_1(t), \mu_{12}$	Mean value	λ_i	Eigen value
(t_1, t_2)			
$w(x, \theta)$	Random process	φ_i	Eigen function
$\bar{w}(x)$	Mean value of random process	C(x1, x2)	Covariance function
M	No. of KL terms	B	Strain displacement matrix
V	Volume of the element	C(X, θ)	Approximated elasticity matrix
D_0	Mean value of elasticity matrix	ν_0	Mean value of Poisson's Ratio
U_0, u, U	Initial velocity	$g(X)$	Limit state function
E_0	Mean value of Young's Moduli	HVI	High velocity impact
G_0	Mean value of shear moduli	LVI	Low velocity impact
IT ₁₃	Interlaminar in-plane shear strength	Pf	Probability of failure
IT ₂₃	Interlaminar out-plane shear strength	H(x)	Polynomial function
Nf	Number of failure sample	B	Polynomial coefficients
N	Total no. of sample	E()	Stationary Gaussian process with zero mean value
R()	Correlation function	$\sigma_g^2(), \sigma$	Variance

$\mu_g(), \mu$	Mean value	CDF	Cumulative distribution function
ψ_1	Changing Pf with respect to mean value	MCI	Matrix cracking initiation
ψ_2	Changing Pf with respect to standard deviation	DI	Damage initiation
MCP	Matrix cracking propagation	DP	Damage propagation