PROBABILISTIC ANALYSIS OF COMPOSITE PLATES UNDER LOW AND HIGH VELOCITY IMPACT

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By

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BE GOOD, DO GOOD, BE ONE

This Ph.D. work is dedicated to My Parents and Teachers

CERTIFICATE

This is to certify that the thesis entitled **“Probabilistic Analysis of Composite Plates under Low and High Velocity Impact"** being submitted by **Mr. Shivdayal Patel** is the report of bonafide research work carried by him under our supervision. This thesis has been prepared in conformity with the rules and regulations of **INDIAN INSTITUTE OF TECHNOLOGY DELHI**. We further certify that the thesis has attained a standard required for the award of a **Doctor of Philosophy** degree of the Institute. The research reported and the results presented in the thesis have not been submitted, in part or full to any other institute or university for the award of any other degree or diploma.

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Shivdayal Patel

ABSTRACT

The low and high velocity impact response of composite targets is an important investigation to assess its reliability for applications to light weight body and vehicle armors. The work has extensive military, aerospace and civil applications. Safety and reliability assessment as per the international standards is one of the basic objectives of the study. The low and high velocity impacts can cause superficial damages such as surfaces scratches which only significantly affect the structural load bearing capability and may be neglected in the analyses. The damage initiates as an intra ply matrix crack due to shear or bending which propagates further into the interface causing de-lamination between dissimilar plies and fiber breakage. This damage evolves with time and adversely affects the mechanical properties and strength of the composite. Since, multiple cracks in the ply are difficult to track, a progressive damage mechanics approach is used to model this failure. The inter ply failure is modeled using cohesive surfaces between the plies. The progressive damage is modeled in terms of damage energy level. When the dame variable reaches a critical value and stiffness is very low the material is not able to sustain any load further and the element does not participate in further calculations. In order to determine the response of the composite under impact, dynamic analysis of composite plate subjected to low and high velocity impacts has been carried out. Damage initiation and propagation failure due to matrix cracking and delamination are investigated for safety criteria for the low velocity impact. Fiber damage initiation and propagation are considered to attain the safety criteria for the S2-glass epoxy composite beam under high velocity impact. The progressive damage model is adopted and implemented in the FE code as a user-defined subroutine (VUMAT). At the onset of study numerical results are obtained using the deterministic progressive damage model and validated against the existing experimental study in the literature. A linear relation exists between impactor velocities (120 m/s to 300 m/s) and numbers of damaged layers for the symmetric cross ply composite beams. This behavior validates the current finding with respect to the published results.

Probabilistic response assessment is strongly recommended by various international codes, especially to achieve challenging design targets for satisfactory service and survival. It is because, the deterministic approach does not account for the natural scatter in the input data. The probabilistic approach explicitly quantifies the extent of safety or probability of failure.

Many a times, design standards recommend a threshold of target reliability for satisfactory design. Probabilistic approach is, all the more, significant to assess the safety of composite armors. The material properties of composites inherently exhibit a scatter or uncertainty due to their inhomogeneity, anisotropic characteristics, brittleness of the matrix and fiber and manufacturing defects. In the conventional approach, deterministic input data is assumed to participate and associated uncertainties are accounted for by using the deterministic factor of safety. This factor of safety too is uncertain to a greater extent. Hence, there is a persistent need to quantify its performance in a realistic fashion, taking in to account the actual mean and statistics data. At present a probabilistic study is carried out to consider the uncertainties of material properties (elastic modulus, Poisson's ratio, shear modulus, strength properties and fracture energy) and initial velocity. In order to take this stochastic variation into consideration the dynamic analysis is performed using the stochastic finite element method. The required discretization of material properties of random nature is done using Latin hyper cube sampling method, which is employed here due to its relative merits. The stochastic progressive damage model is developed and implemented in the SFE code NESSUS as a user-defined subroutine (VUMAT). The randomly generated sets of properties are used in the stochastic FE analysis to provide the statistics of critical stresses at strategic locations. These stresses and random strengths properties are used for the comparative failure assessment using different failure criteria. On substitution of these critical stresses and respective random strength values, in terms of their statistical characteristics and distributions in the limit state function, a joint probability of failure (Pf) of each ply is obtained. This is numerically evaluated using the Gaussian process response surface method (GPRSM). Multi dimensional integral for determining Pf is calculated by using Gaussian process response surface method (GPRSM). Monte Carlo simulation (MCS) for evaluating the same integral required 10,000 cycles to reach a converged value of Pf and takes almost 40 times more computational time than GPRSM for the same accuracy. This procedure is adopted to investigate the Pf for low velocity impact (LVI) and high velocity impact (HVI) based on damage initiation and propagation models. Comparative study of the probability of failure is carried out using different fiber damage initiation and propagation criteria. The optimum ply arrangement is studied for fiber failure initiation and propagation of composite beams for simply supported and clamped-clamped boundary conditions. System Pf and

sensitivity analysis are also investigated for a composite plate under low and high velocity impact.

The target reliability may be achieved for an optimal combination of the impactor mass and velocity. There is a possibility of underestimation of the peak contact force and displacement by 10.7% and 11.03%, respectively, if the scatter in the material properties and impactor velocity are not considered and mean values are used for calculating response. A 3D stochastic finite element analysis based design optimization of laminated composite beams under high velocity impact is performed to determine the lay-up arrangement having a lower probability of failure (Pf). Comparative study has been carried out, in a novel way, in terms of probability of failure for different ply arrangements, boundary conditions using various failure criteria. Comparative study of different fiber failure initiation criteria is studied and found that a maximum stress criterion is more conservative than the Yen and Hashin failure criterion. There is a possibility of underestimation of maximum areas of the fiber damage and matrix damage by 62.5% and 38.05%, respectively, if the scatter in the material properties and impactor velocity are not considered and mean values are used for calculating response. For the parallel and mixed systems Pf due to fiber damage propagation for clamped-clamped and simply supported BCS of anti-symmetric cross ply arrangement (Case-II) is lesser than that for other ply arrangements, namely, symmetric cross ply (Case-I), symmetric angle ply (Case-III) and anti-symmetric angle ply (Case-IV). Sensitivity based design optimization is carried out for all ply arrangements and different boundary conditions. This is an important input for the stochastic design optimization. The study provides a realistic procedure for design optimization that duly considers the probabilistic scatter in all the properties involved in the analysis and design of composite body armors.

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NOMENCLATURE

$\sigma_{ij,}$ $[\sigma]$	Stress at tensor	Y_t	Transverse tensile strength
ρ	Density	Y_c	Transverse compressive strength
$\ddot{u}_i$, $\ddot{u}_B$	Acceleration components	σ_{33}^{t}	Normal Transverse tensile stress
u_i , u_B	Displacement components	σ_{33}^{c}	Normal Transverse compressive stress
b_i	Body force components	Z_t	Normal Transverse tensile strength
ε_{ij} , $[\varepsilon]$	Strain at tensor	Z_c	Normal Transverse compressive strength
Fci, Fc (t)	Contact force components	τ_{12} , τ_{13}	In-plane shear stresses
mi	Mass of impactor	τ_{13}	Out of plane shear stresses
M	Mass of plate	T_{12} , T_{13}	In-plane shear strengths
K , C_0	Stiffness of the plate	T_{23}	Out of plane shear strength
Ω	Effective damage tensor	σ_n^n	Normal stress component
d_f^t	Tensile fiber damage	σ_t^t	Tangential stress component
d_f^c	Compressive fiber damage	σ_n^t	Stress component normal to tangential direction
d_m^t	Tensile matrix damage	σ_n^l	Stress component normal to longitudinal direction
d_m^c	Compressive matrix damage	T_{23}^A	Transverse shear strength
d_f	Fiber damage	μ_n^t	Frictional coefficient normal and tangential direction
d_m	Matrix damage	μ_n^l	Frictional coefficient normal and longitudinal direction
$\dot{\sigma}$	Incremental stress tensor	$\varepsilon_{f,1}^t$	Final fiber failure in tension
$\dot{\varepsilon}$	Incremental strain tensor	$\varepsilon_{0,1}^t$	Initial fiber failure in tension
$\dot{\Omega}$	Incremental damage tensor	ε_{11}	Longitudinal strain
E_{ij}	Young’s Modulii	$\dot{\varepsilon}_{11}$	Incremental longitudinal strain
G_{ij}	Shear Modulii	$\varepsilon_{f,1}^c$	Final fiber failure in compression
v_{ij}	Poisson’s ratio	$\varepsilon_{0,1}^c$	Initial fiber failure in compression

C	Degraded stiffness matrix
$\dot{d}_f$	Incremental fiber damage
$\dot{d}_m$	Incremental matrix damage
D	Damage tensor
ε	Strain at any point ‘t’
ε_0	Initiation strain
ε_f	Final failure strain
Gc	Fracture energy
σ^0	Initial strength
L	Characteristics Length
T_{ij}	Shear strength
σ_{11}^t	Longitudinal tensile stress
σ_{11}^c	Longitudinal compressive stress
X_t	Longitudinal tensile strength
X_c	Longitudinal compressive strength
σ_{22}^t	Transverse tensile stress
σ_{22}^c	Transverse compressive stress
q_{ij}	Damage coupling coefficients
δ_n^0	Initial normal separation
δ_t^0	Initial tangential separation
δ_s^0	Initial shear separation
δ_n^f	Final normal separation
δ_t^f	Final tangential separation
δ_s^f	Final shear separation
G_n , G_{IC}	Normal fracture energy
G_s , G_{IIC}	Shear facture energy
G_t , G_{IIIC}	Tangential fracture energy
G_n^c	Critical Normal fracture energy
ε_{22}	Transverse strain
$\dot{\varepsilon}_{22}$	Incremental transverse strain
$\varepsilon_{m,2}^t$	Final matrix failure in tension
$\varepsilon_{0,2}^t$	Initial matrix failure in tension
$\varepsilon_{m,2}^c$	Final matrix failure in compression
$\varepsilon_{0,2}^c$	Initial matrix failure in compression
λ_r^t	Resultant shear strain
$\dot{\lambda}_r^t$	Incremental resultant strain
λ_{12} , λ_{13}	In plane shear strains
λ_{23}	Out of plane shear strain
r_i	Damage thresholds
< >	Maclory Bracket
ε_{33}	Out of plane strain
T_{33}	Out of plane strength
M, m	Material constant
ϖ_j	Damage variables
$\dot{\varpi}_j$	Incremental damage variables
ϖ_1	Fiber damage in tension
ϖ_3	Fiber damage in compression
ϖ_2	Matrix damage in tension
ϖ_4	Matrix damage in compression
ϖ_5	Fiber crushing damage
ϖ_6	Matrix damage in shear
t_n	Normal traction
t_s	Shear traction
t_t	Tangential traction
σ_n	Normal strength
σ_t	Tangential strength

G_s^c	Critical Shear facture energy	σ_s	Shear strength
G_t^c	Critical Tangential fracture energy	C	Transition temperature
A, B	Material Hardening constants	μ	Frictional coefficient
$\dot{\varepsilon}_0$	Strain rate	G_f^t	Fiber fracture energy in tension
θ_m	Melting temperature	G_f^c	Fiber fracture energy in compression
N	Normal strength	G_m^t	Matrix fracture energy in tension
S	Shear strength	G_m^c	Matrix fracture energy in compression
T	Tangential strength	K_{nn}	Normal stiffness
1M	8.66 g of impactor mass	K_{ss}	Shear stiffness
X(t)	Random fields	K_{tt}	Tangential stiffness
$\alpha_1(t)$, μ_{12} (t_1,t_2)	Mean value	λi	Eigen value
w(x, θ)	Random process	φ_i	Eigen function
$\bar{w}(x)$	Mean value of random process	C(x1, x2)	Covariance function
M	No. of KL terms	B	Strain displacement matrix
V	Volume of the element	C(X,θ)	Approximated elasticity matrix
D_0	Mean value of elasticity matrix	ν_0	Mean value of Poisson's Ratio
U_0, u, U	Initial velocity	g(X)	Limit state function
E_0	Mean value of Young's Modulii	HVI	High velocity impact
G_0	Mean value of shear modulii	LVI	Low velocity impact
IT_{13}	Interlaminar in-plane shear strength	Pf	Probability of failure
IT_{23}	Interlaminar out-plane shear strength	H(x)	Polynomial function
Nf	Number of failure sample	B	Polynomial coefficients
N	Total no. of sample	E()	Stationary Gaussian process with zero mean value
R()	Correlation function	$\sigma_g^2()$, σ	Variance

$\mu_g()$, μ	Mean value	CDF	Cumulative distribution function
ψ_1	Changing Pf with respect to mean value	MCI	Matrix cracking initiation
ψ_2	Changing Pf with respect to standard deviation	DI	Damage initiation
MCP	Matrix cracking propagation	DP	Damage propagation