

STUDIES IN PARAMETER ESTIMATION USING  
WALSH FUNCTIONS APPROACH

ANAND SWARUP SAXENA

Thesis submitted to the  
Indian Institute of Technology, Delhi  
for the award of the Degree of  
DOCTOR OF PHILOSOPHY

DEPARTMENT OF ELECTRICAL ENGINEERING  
INDIAN INSTITUTE OF TECHNOLOGY, DELHI  
NEW DELHI - 110016

NOVEMBER, 1988

REVISED APRIL, 1991

C E R T I F I C A T E

This is to certify that the thesis entitled, "STUDIES IN PARAMETER ESTIMATION USING WALSH FUNCTIONS APPROACH", being submitted by Mr. Anand Swarup Saxena for the award of the degree of DOCTOR OF PHILOSOPHY to the Indian Institute of Technology, Delhi, is a record of the original bonafide research work carried out by him under our guidance and supervision. The results contained in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

*V.S. Rajamani*  
Prof. V.S. Rajamani 3/11/88

*Anand Swarup Saxena*  
Dr. A.N. Jha 3.11.88

Department of Electrical Engineering,  
Indian Institute of Technology, Delhi,  
New Delhi -110016

C E R T I F I C A T E

This is to certify that the thesis entitled, "STUDIES IN PARAMETER ESTIMATION USING WALSH FUNCTIONS APPROACH", submitted by Mr. Anand Swarup Saxena for the award of the degree of DOCTOR OF PHILOSOPHY to the Indian Institute of Technology, Delhi, in Nov. 1988, has been revised and is being resubmitted.

*V.S. Rajamani*  
Prof. V.S. Rajamani 24/1/91

*Anand Swarup Saxena*  
Dr. A.N. Jha, 2.4.91

Department of Electrical Engineering,  
Indian Institute of Technology, Delhi  
New Delhi-110016

DEDICATED TO  
PARENTS,  
SUMAN, MY WIFE  
AND  
AKHIL, ANURAG AND AMIT, MY SONS

## A C K N O W L E D G E M E N T S

I am extremely grateful to my supervisors, Professor V.S. Rajamani and Dr.A.N.Jha, for suggesting this interesting area of research. I express my sincere thanks for their invaluable guidance, constant encouragement and critical review throughout the course of this work.

My thanks are due to my research colleagues Dr.V.Ranganathan and Mr. P.G. Rao for their valuable assistance at various stages of this work.

I am thankful to Prof. K.K.Shrivastava, Principal, M.I.T.S. Gwalior-5 for sponsoring me under Q.I.P. Scheme and encouragement given at various stages. I acknowledge with thanks the cooperation received in the last stage of this work from Dr.R.D.Gupta (Prof. and Head) and my colleagues of Electronics Department.

I express my appreciation and thanks to Mr. J.N.Saini, for efficient typing of the thesis.

Finally, I am grateful to my parents for their constant inspiration, to my wife Suman and sons, Akhil, Anurag and Amit for their patience, understanding and encouragement during the course of my research.

Anand Swarup Saxena

## A B S T R A C T

The thesis addresses the problem of parameter estimation in lumped and distributed parameter systems employing Walsh functions. The method used is based on the use of orthogonal functions (Walsh functions) in reducing lumped and distributed parameter systems models to algebraic equations. Greville's algorithm is used to develop a recursive algorithm to get an estimate of the parameters of the model. The recursive algorithms, developed for various types of system considered, have the features of computational simplicity and finite memory requirement. The algorithm can be used to identify the unobservable systems also. These algorithms enable the estimation of initial and boundary conditions along with the unknown parameters.

In all six problems have been considered in this thesis. Firstly, a recursive least-squares estimation algorithm has been developed for evaluating the coefficients of Walsh functions expansion of a signal. This algorithm is extended to cover the two-variable case. In the second problem, parameter estimation procedures for linear distributed parameter systems of first-order and second-order have been presented using recursive algorithms.

Next two problems consider the parameter estimation in bilinear lumped systems, a class of non-linear lumped systems with fractional powers in non-linearity and non-linear distributed parameter systems. Recursive algorithms have been developed for estimation procedure of all these problems. Appropriate numerical examples have been included to illustrate the use of algorithms.

The last two problems deal with the parameter estimation in linear lumped time-varying system and linear time-varying distributed parameter systems. Estimation procedures for these systems are based on the recursive algorithms developed.

# C O N T E N T S

	Page
ACKNOWLEDGEMENTS	
ABSTRACT	
CHAPTER-1 INTRODUCTION ...	1
1.1 Parameter Estimation in Lumped and Distributed Parameter Systems ...	3
1.2 Scope of the Thesis ...	5
1.3 The Problem of Parameter Estimation ...	6
1.3.1 General Model Equations for Distributed Parameter Systems ...	7
1.3.2 Parameter Estimation Problem ...	9
1.3.3 Model used in the Thesis ...	9
1.4 Outline of the Thesis ...	10
CHAPTER-2 LITERATURE SURVEY	
2.1 Distributed Parameter Systems Theory ...	13
2.1.1 System Identification and Parameter Estimation ...	13
2.2 Review of Orthogonal functions and their applications ...	15
2.2.1 Walsh functions ...	15
2.2.2 Block-pulse functions ...	24
2.2.3 Laguerre Polynomials ...	31
2.2.4 Legendre Polynomials ...	34
2.2.5 Poisson Moment functionals ...	38
2.2.6 Chebyshev Polynomials ...	40

CHAPTER-3	ON-LINE LEAST-SQUARES METHOD OF ESTIMATING THE COEFFICIENTS OF THE WALSH FUNCTIONS EXPANSION OF A FUNCTION		
3.1	Introduction	...	43
3.2	An Introduction to Walsh Functions	...	44
	3.2.1 Properties of Walsh Functions	...	48
	3.2.2 Operational Matrix of Integration...		53
3.3	Recursive Least Squares Estimation Algorithm for Coefficients of Walsh functions expansion-Single Variable Case	...	54
3.4	Numerical Examples-Single Variable case.	...	56
	3.4.1 Effect of White noise	...	59
3.5	Recursive Least Squares Estimation Algorithm for coefficients of Walsh functions expansion-Two Variable case.	...	60
3.6	Numerical Example -Two Variable Case.	...	63
	3.6.1 Effect of White noise	...	63
3.7	Conclusion	...	63
CHAPTER-4	PARAMETER ESTIMATION IN LINEAR DISTRIBUTED PARAMETER SYSTEMS VIA WALSH FUNCTIONS - RECURSIVE APPROACH	...	67
4.1	Introduction	...	67
4.2	Problem Formulation	...	69
4.3	Mathematical Preliminaries	...	69
4.4	The Parameter Estimation Method	...	73
	4.4.1 First-order distributed para- meter systems	...	73
	4.4.2 Second-order distributed para- meter systems	...	76

4.5	Numerical Example - First Order System ...	80
4.5.1	Effect of White noise ...	80
4.6	Comparison of Simulation Results ...	83
4.7	Conclusion ...	84
CHAPTER-5	PARAMETER ESTIMATION ALGORITHMS FOR BILINEAR AND NONLINEAR SYSTEMS USING WALSH FUNCTIONS-RECURSIVE APPROACH ...	85
5.1	Introduction ...	85
5.2	Problem Formulation ...	87
5.3	Mathematical Preliminaries ...	88
5.4	Estimation of Parameters in Bilinear Systems ...	90
5.5	Numerical Example-Bilinear System ...	93
5.5.1	Effect of White noise ...	96
5.6	Estimation of Parameters in Non-linear Systems. ...	97
5.7	Numerical Example - Nonlinear System ...	99
5.7.1	Effect of White noise ...	102
5.8	Comparison of Simulation Results ...	103
5.9	Conclusion ...	104
CHAPTER-6	PARAMETER ESTIMATION IN NON-LINEAR DISTRIBUTED PARAMETER SYSTEMS VIA WALSH FUNCTIONS-RECURSIVE APPROACH ...	105
6.1	Introduction ...	105
6.2	Problem Formulation ...	106
6.3	Mathematical Preliminaries ...	106

6.4	The Parameter Estimation Process	...	108
6.5	Numerical Example	...	114
	6.5.1 Effect of White noise	...	116
6.6	Comparison of Simulation Results	...	117
6.7	Conclusion	...	118
CHAPTER-7	PARAMETER ESTIMATION IN LUMPED LINEAR TIME VARYING SYSTEMS AND DISTRIBUTED LINEAR TIME VARYING SYSTEMS VIA WALSH FUNCTIONS - RECURSIVE APPROACH	...	119
7.1	Introduction	...	119
7.2	Problem Formulation - Lumped Linear Time-Varying Systems	...	121
7.3	Mathematical Basis for the Proposed Method	...	122
7.4	Estimation Procedure for Lumped Linear Time-varying systems	...	125
7.5	Problem Formulation for Distributed Linear Time-Varying Systems	...	132
7.6	Estimation Procedure for Distributed Linear Time-Varying Systems	...	132
7.7	Conclusion	...	137
CHAPTER-8	SUMMARY AND SUGGESTIONS FOR FURTHER RESEARCH	...	139
8.1	Summary of Results	...	139
8.2	Suggestions for Further Research Work	...	141
	R E F E R E N C E S	...	143
	A P P E N D I X	...	155
	CURRICULUM VITAE	...	159