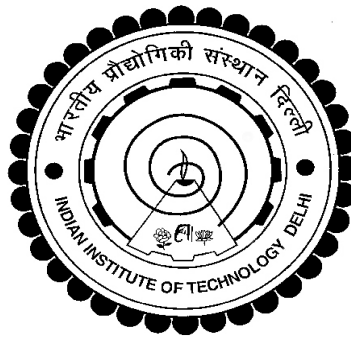


STABILITY OF A WEAK CONFINED WAKE BEHIND A CYLINDER IN FULLY DEVELOPED TURBULENT CHANNEL FLOW

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NOVEMBER 2016**

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by

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Submitted

in fulfilment of the requirements of the degree of **DOCTOR OF PHILOSOPHY**

to the



INDIAN INSTITUTE OF TECHNOLOGY DELHI
HAUZ KHAS, NEW DELHI – 110016
November 2016

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**DEDICATED TO MY PARENTS AND
TEACHERS**

CERTIFICATE

This is to certify that the thesis titled “***STABILITY OF A WEAK CONFINED WAKE BEHIND A CYLINDER IN FULLY DEVELOPED TURBULENT CHANNEL FLOW***” being submitted by Mr.T. VIJAYA KUMAR to the Department of Applied Mechanics, Indian Institute of Technology Delhi, for the award of degree of **DOCTOR OF PHILOSOPHY** is a record of bonafide research work carried out by him under our supervision. This thesis has been prepared in conformity with the rules and regulations of Indian Institute of Technology Delhi, New Delhi, India. We further certify that the thesis has attained a standard that makes it eligible for a PhD degree of the Institute. The research reported and results presented in the thesis have not been submitted, in part or full to any other Institute or University for the award of any degree or diploma.

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ACKNOWLEDGEMENTS

I am deeply indebted to my thesis supervisors Professor P.K. Sen, Professor Srinivas V. Veeravalli and Dr. Munendra Kumar for their continued motivation, constant encouragement and remarkable guidance throughout the course of this work. I wish to express gratitude from the deepest core of my heart to Professor P. K. Sen and professor Srinivas V. Veeravalli for accepting me as their PhD student and the support and invaluable guidance they rendered throughout. My heartfelt thanks to Dr. Munendra Kumar, for many useful discussions and guidance especially in numerical work.

I express my sincere thanks to Professor Padma Vasudevan Sen and Dr. Sriram Hegde for their support, encouragement, advice and care at every stage of this work till completion. It gives me immense happiness to acknowledge Ms. Sudha Sen, mother of Prof. P. K. Sen, for her interest and blessings in completing my thesis on time. My sincere thanks and acknowledgement to Dr. Murali R. Cholemari for the technical discussions and help he extended in completing this work successfully. My heartfelt thanks to Dr. Ganapati Joshi for his moral support and technical interactions, which had an accelerating effect on carrying forward my research work.

I express my gratitude to Mr.Chandahas S. Shet and Mr. Sheikh Nasiruddin for their technical and moral support throughout the experimental work. I acknowledge the help with gratitude of the staff of Gas Dynamics laboratory especially Mr.Suresh Sharma, Mr.Kunj Bihari, Mr.R P Bhogal and Mr. Nand Lal. I express my gratitude to all my colleagues at DTU who were a source of constant inspiration during my research work.

It is indeed a pleasure to express gratitude towards the moral support provided by my parents and in-laws. My wife Mrs. R. Rani Hema Latha, sons V.Satish and V.Ravi deserve a special mention here who had to endure my delayed homecoming during the course of my work. They whole heartedly tolerated the inconveniences due to my preoccupation with the thesis work and sacrificed their fun and entertainment to allow me to complete my work in time and cooperated with me in all possible ways in a matured way. I wish to express heartfelt gratitude to my late father in law Sh. T. Ramaiyan whose constant encouragement and moral support till his last breath to submit the thesis in time.

Last but not the least, I am grateful to all those who have helped me directly or indirectly during the course of this work.

28 October 2016,

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ABSTRACT

The basic task of hydrodynamic stability theory is to explain all possible phases of transition from laminar flow to turbulent flow in various dynamic systems of fluid mechanics. Nevertheless turbulent flow problems are still considered unresolved. The problem of predicting the stability characteristics of wake flow is of great theoretical and practical importance, because these flows occur very frequently in engineering systems, especially in environmental, aeronautical, civil and industrial engineering applications, due to possible resonance of the Strouhal frequency with structural resonance modes. The history of the problem is discussed in brief, including organised structures in laminar and turbulent wakes and hydrodynamic stability analysis in some cases. Brown & Roshko (1974), and Roshko (1953, 1992) for example, performed experiments on turbulent mixing layer and wakes. Organised, coherent structures and other processes like vortex pairing, triplet and quadruplet formation were observed in these experiments, which were very similar to the ones observed in the laminar mixing layer. These breakthrough works provide a strong motivation to apply stability theory for the understanding of major processes and changes happening in fully developed turbulent free shear flows

The main difference between wakes and other flows, like wall bounded flows, and other free shear flows, is the presence of stream wise travelling vortices, commonly called Karman Vortex Street, which is mainly caused by the internal instability of the base flow which is the unique feature in these flows even in the turbulent state. Compared with wall bounded flows, free shear flows are highly unstable due to the presence of an inflection point in the velocity profile and the associated inviscid instability.

The motivation for the study of instability of turbulent wake flow in a confined turbulent channel was multi-fold. First, the instability of confined wake flows has not been studied much. Second, confined wakes are found to retain their mean velocity profile for a considerable downstream distance. Third, wakes have two points of inflection, one each on either side of the centre line. The basic aim of the present study was to investigate the correlation between the turbulence in the wake region and the inflection points in the wake region using stability theory. The wake behind a cylinder of diameter d in a turbulent channel with half width h , was found to be a weak confined wake when $d/h \approx 0.1$. Thus, $d/h = 0.1$ was chosen for the present work. Detailed investigation has been done and results have been obtained by introducing organized disturbances in the wake and tracking these downstream. Eigenfunctions were obtained by two methods viz. i) Phase locked averaging and ii) Spectral analysis. Theoretical results were obtained by solving the Orr-Sommerfeld equation by numerical methods.

Based on the experimental results and upon comparing these results with theoretical ones, it was found that both theoretical and experimental results are fairly well matched. Results also show that the additional turbulence generated in the wake sustains in the downstream direction and its decay is well in agreement with stability theory predictions.

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LIST OF SYMBOLS

A	Spectral density
A_0	Spectral density at the first location (at $X=10\text{mm}$)
A_i	Coefficients in OS equation
A_{ij}	Coefficient Matrix
A_l	Spectral density at the first location in the absence of cylinder
C	Constant
c	Wave speed, complex
c_i	Wave speed , imaginary, temporal growth rate
c_r	Wave speed, real
c_g	Group velocity
d	diameter of the cylinder
$E = \frac{\varepsilon}{\nu}$	Non dimensionalised Eddy viscosity
f	Frequency
F_n	Natural frequency of the cylinder
F_{1l}	Longitudinal spectrum
$CO(f)$	Cross-spectrum
g_i	Auxiliary function

H	Channel half width
h	Channel full width
$L(\alpha)$	Differential operator
P	Instantaneous Pressure
p	Pressure fluctuation
$\tilde{p}$	Pressure component due to organised disturbance
Re	Reynolds number
R_{crit}	Critical Reynolds number
R_{flow}	Flow Reynolds number
S_t	Strouhal number
t	time
u, v, w	Velocity fluctuation
$\hat{u}(y)$	Amplitude of fluctuation velocity from Orr-Sommerfeld (OS) solution
$\tilde{u}$	rms velocity component due to organised disturbance
U_{rms}	rms velocity of turbulence in the presence of cylinder
U_{rms0}	rms velocity of turbulence in the absence of cylinder (free stream turbulence)
$u_r = \sqrt{(U_{rms}^2 - U_{rms0}^2)}$	Excess rms velocity of turbulence in the wake
u_*	Friction velocity

U_o	Centreline velocity
U	Instantaneous velocity, Shoulder velocity
$\overline{U}$	Mean velocity
V	Instantaneous transverse velocity
$\bar{V}$	Mean Transverse velocity
V_s	Voltage across speaker
x	Down stream distance from the cylinder
X	Down stream distance from the slot
$x^* = X/\lambda$	Downstream distance normalised
$y = \frac{y_{dim}}{H}$	Distance across the channel from the wall
δ	Central difference operator
θ	Phase angle
ε	Eddy Viscosity
λ	Half width of Gaussian distribution , distance from inflection point to centre of channel
α	Wave number
α_i	Wave number, imaginary, spatial growth rate
α_r	Wave number, <i>real</i>
$\beta = 2\pi f$	Circular frequency
β_r	frequency, <i>real</i>

$\beta_i = \alpha_r c_i$	frequency, imaginary, temporal growth rate,
μ	Dynamic viscosity
ρ	Density
ψ	Disturbance stream function
ϕ	Eigenfunction of disturbance stream function
$\emptyset_j$	Gauss-Noumerov transformation
τ_w	Wall shear stress
ν	Kinematic viscosity

Superscript

'	differentiation with respect to y
+	Normalization with inner variables

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