

Near-Semirings

Theory and Application

By

K. V. Krishna

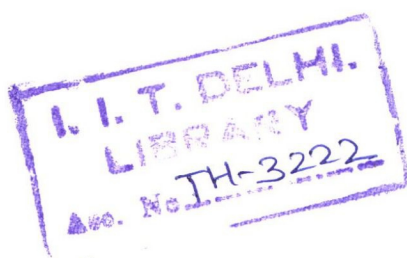
Department of Mathematics

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of the degree of Doctor of Philosophy*

to the



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Certificate

This is to certify that the thesis entitled *Near-Semirings: Theory and Application* submitted by *Mr. K. V. Krishna* to the Indian Institute of Technology Delhi, for the award of the Degree of Doctor of Philosophy, is a record of the original bona fide research work carried out by him under my supervision and guidance. The thesis has reached the standards fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted in part or full to any other university or institute for the award of any degree or diploma.

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Dr. Niladri Chatterjee
Supervisor

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Abstract

The thesis aims at further development in the theory of near-semirings along with some applications to theoretical computer science. The study discusses some of the issues concerning theory of radicals, representation theory and structure theory of near-semirings. Also, as an application, the thesis investigates the role of near-semirings in linear sequential machines.

In order to study the internal structure of radicals of near-semirings, introduced by van Hoorn, the thesis introduces the concepts: ν -modular left ideal, quasi-regularity and prime ideals in near-semirings, and studies various properties of radicals. Both in classical algebraic and categorical approaches the thesis observes that S -semigroups are representations of near-semirings. Further, this work addresses some approximations to find a suitable category in which a given near-semiring is primitive. To study some structural properties of near-semirings, this thesis extends Holcombe's holonomy decomposition of near-rings to near-semirings, employing the techniques of Eilenberg. In view of applications of near-semirings, the thesis introduces an α -radical of affine near-semirings and finds a necessary condition to test the minimality of generalized linear sequential machines.

Contents

Certificate	i
Acknowledgements	iii
Abstract	v
Introduction	1
1 Fundamental Notions	9
1.1 Near-Semirings	10
1.2 S -Semigroups	13
1.3 Left Ideals & Ideals	18
1.4 Modular Left Ideals	23
2 Theory of Radicals	27
2.1 Types of S -Semigroups & Radicals	28
2.2 ν -Modular Left Ideals, $\nu = 0, 1, 2$	32
2.3 Quasi-Regularity & Prime Ideals	35
2.4 Descending Chain Condition	38

3	Representations and Approximations	41
3.1	Algebraic Representation	42
3.2	Categorical Representation	43
3.3	Approximation Theorems	49
4	Holonomy Decomposition	55
4.1	Transformation Semigroups	56
4.2	$\mathfrak{M}_{G^0}(\Gamma)$ and Its Skeleton	58
4.3	Main Result	63
5	Affine Near-Semirings	69
5.1	Fundamental Notions	70
5.2	The α -Radical	72
6	Generalized Linear Sequential Machines	77
6.1	State Function of $\mathcal{M}$	78
6.2	Syntactic Near-Semiring	80
6.3	Minimization	83
	Bibliography	87
	Index	93
	Bio-Data	95