

ON THE APPROXIMATE ANALYSIS OF HYPERBOLIC PARABOLOID SHELL ROOFS
BY THE METHOD OF WEIGHTED RESIDUALS
(VOLUME I)

by

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A B S T R A C T

The thesis deals with the approximate analysis of shallow doubly curved shells by the methods of weighted residuals, (MWR), in particular the methods of collocation, Galerkin and least squares, for the purpose of designing them in reinforced concrete. A part of the work is concerned with the analytical and experimental investigation of a non-symmetric hyperbolic paraboloid shell proposed for roofing an auditorium at a nearby university.

After a review of the relevant literature, the shallow shell equations and the principles of the MWR are presented and the main features of the numerical formulation utilised are discussed.

The convergence of the solution by the Galerkin method is studied. Suitable indices of errors are suggested to estimate the accuracy of the solutions at any stage.

To remove the arbitrariness in the selection of the matching points, modified collocation procedures and iterative techniques are tried with some success. Optimisation methods are used to remove the uncertainty. Suitable object functions for minimisation are discussed.

The least squares method is found to give unsatisfactory results. Improvement is obtained by adapting a weighted least squares approach. A convenient method which incorporates the advantages of weighted least squares process and the simplicity of the collocation is illustrated.

The Galerkin method with iterative procedure is used for the geometrically non-linear analysis. The convergence and the difference between the linear and the non-linear solutions are illustrated.

The effects of initial imperfections, and settlement of supports, that are unavoidable in reinforced concrete shells, on the stress resultants are discussed with reference to particular examples.

The eigen value problems of estimating the natural frequencies and the buckling loads are analysed.

A non-symmetric hyperbolic paraboloid shell proposed for roofing an auditorium is analysed. The detailed experimental investigation of a scale model of the auditorium shell is presented. The reasons for the discrepancies between the theoretical and experimental results are discussed.

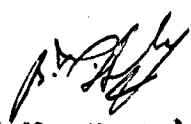
Four sets of design tables for shells are presented.

Conclusions drawn from the study are summarised.

CERTIFICATE

This is to certify that the thesis entitled 'On the Approximate Analysis of Hyperbolic Paraboloid Shell Roofs by the Method of Weighted Residuals' being submitted by Shri V. Raja Rao to the Indian Institute of Technology, Delhi for the award of Degree of Doctor of Philosophy in Civil Engineering, is a record of bonafide research work carried out by him. Shri V. Raja Rao has worked under my guidance and supervision and has fulfilled the requirements for the submission of this thesis, which to my knowledge has reached the requisite standard.

The results contained in this thesis have not been submitted, in part or in full, to any other University or Institute for the award of any degree or diploma.



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