

STUDIES IN LAMINAR DISPERSION

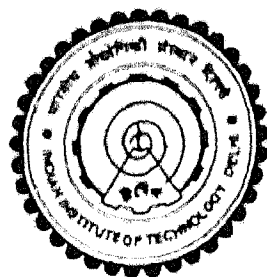
By

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CERITIFICATE

This is to certify that this thesis entitled **Studies in Laminar Dispersion** being submitted by Mr. Om Prakash to the Indian Institute of Technology, Delhi for the award of the degree of **Doctor of Philosophy** is a record of bona fide research work carried out by him. **Mr. Om Prakash** has worked under our supervision and has fulfilled the requirement for the submission of thesis.

The results contained in this thesis have not been submitted in part or in full to any other University or Institute for the award of any degree or diploma.



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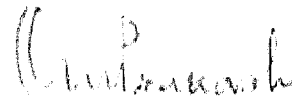
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[Om Prakash]

ABSTRACT

The study of dispersion contributes substantially in designing and better understanding of the performance of chemical reactors and heat exchangers etc. The analysis of dispersion is applicable in many diverse fields of science and engineering such as chemical reaction engineering, mass transfer, secondary and tertiary recovery of crude oil, soil physics, mining, physiological applications involving measurement of flow rates in combined systems such as human body, flow rate measurement in large open streams and pipe lines and dispersion of pollutants in water streams.

Taylor (1953) analysed the problem of dispersion of material in a straight tube and his one dimensional plug flow model can be used to represent a two dimensional situation. He gave the expression for dispersion coefficient and estimated the conditions under which his analysis would be expected to hold. He provided experimental support to his theory of dispersion and thus established the basic concept of the dispersion theory. The simplicity and elegance of the theory and the importance of the results of the dispersion theory have attracted the attention of many researchers and a considerable work has been done since the inception of the theory of dispersion. The situations which have been examined include turbulent flow in straight tube, laminar flow in parallel, convergent and divergent plates, laminar flow in helical coils, effect of stagnant pockets and velocity profile (influenced by geometrical shapes of conduit).

The present study analysed the result of axial dispersion phenomenon on following situations:

a. **Laminar Dispersion in Porous Media**

The main aim of the study was to investigate the laminar dispersion of a solute in the tube packed with a porous material of permeability K . The fully developed flow through a porous medium offers two resistances i.e. Darcy resistance and viscous resistance. The expression for fully developed velocity profile has been obtained by solving the momentum equation. A dimensionless permeability parameter has been defined and its effect on velocity distribution, mean velocity and ratio of mass velocity of packed tube to mass velocity of empty tube has been studied. The velocity profile expression was used in the mass balance equation to find the effective diffusion coefficient. The effect of permeability parameter on effective diffusion coefficient has been studied and it has been shown that under limiting case the effective diffusion coefficient expression reduces to the standard Taylor's (1953) effective dispersion coefficient for straight tube.

b. **Longitudinal Dispersion of Sinusoidally varying reactive species in Laminar flow through a tube**

The problem of dispersion of a reacting contaminant injected sinusoidally in a fluid flowing through a tube has been studied. The Taylor's (1953) dispersion theory has been applied to obtain reaction dependent effective diffusion coefficient. The effect of rate constants and the pulsation of input of the contaminant on the decay distance has been investigated. It was found that for a given reaction rate constant the decay distance increases with decrease in the value of dimensionless frequency parameter.

c. **Longitudinal Dispersion in Slowly Varying Axisymmetric Tube**

The concept of slowly varying flow forms the basis of a large class of problems in fluid mechanics and has applications in many areas of biomedical engineering viz. to determine the shear stress distribution on the wall of an arteriosclerotic blood vessel.

An attempt was made to study the phenomenon of dispersion in a tube of slowly varying cross-section within the framework of Taylor dispersion model. An expression for dispersion coefficient has been derived in terms of Peclet Number and a small parameter ϵ . The variation of dispersion coefficient with Reynolds number, Schmidt number and parameter ϵ has been studied.

d. **Residence Time-Distribution (RTD) for Diffusion free Laminar flow in a Tube with Pulsating Flow**

Practically no theoretical analysis or experimental informations are available on the influence of pulsations produced by metering pump on the residence time distribution under the influence of negligible molecular diffusion.

The step response experiments were carried out in straight tube using diethylene glycol and aqueous solution of 80% DEG. The dimensionless frequency parameter $\left\{ \Omega = a \sqrt{\omega / \nu} \right\}$ was varied from 0.73 to 3.26. A gradual narrowing of RTD was obtained with increase in Ω . A relationship between the time at which the tracer appears at the outlet end of the tube ($\bar{\theta}_{\min}$) and the dimensionless frequency parameter was determined. A correlation similar to Danckwert type RTD was

derived and a comparison of experimental data with that of correlation has also been made.

The step response experiments were carried out in coils under significant molecular diffusion conditions. The effect of variation of dimensionless frequency parameter on $D_e / \bar{U}L$ has been studied. A relationship between $D_e / \bar{U}L$ and Ω was obtained. The experimental results were compared with Joshi et al.'s (1983) results for gases for different dimensionless frequency parameters. The present experimental data were also compared with the experimental results of Trivedi and Vasudeva (1975) for steady flow in coils and Nigam and Vasudeva's (1976) experimental findings under pulsatile flow conditions.

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