DAMPED STOCHASTIC SYSTEM DRIVEN BY COLORED NOISE : ANALYTICAL AND DIGITAL SIMULATION STUDIES

by

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I dedicate this thesis to

my beloved parents

whose affection is my inspiration

CERTIFICATE

This is to certify that the thesis entitled "Damped stochastic system driven by colored noise: Analytical and digital simulation studies", being submitted by Chitralekha Mahanta to the Department of Electrical Engineering, Indian Institute of Technology, Delhi, for the award of the degree of Doctor of Philosophy, is a bonafide research work carried out by her under our supervision and guidance. The results contained in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

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Abstract

The thesis concentrates on solving the problem of nonlinear non-Markovian stochastic process associated with a damped stochastic system driven by colored noise. Stochastic nonlinear phenomena are encountered in a number of fields including physics, chemistry, engineering, biology, and humanities. Brownian motion of a particle in a general one-dimensional nonlinear potential driven by a Gaussian Markovian noise acts as a simple mathematical model for a wide variety of systems and phenomena encountered in the above mentioned fields. Such Brownian motions are governed by stochastic differential equations (SDEs) given by, $\ddot{x} = -\gamma\dot{x} - \frac{dU(x)}{dx} + \xi(t)$, where x is the position of the Brownian particle with overdots representing derivatives with respect to time, γ is the damping coefficient, $U(x)$ is the general nonlinear potential and $\xi(t)$ is the colored noise defined by $\langle \xi(t)\xi(s)\rangle = \frac{D}{\tau}\exp(-|t-s|/\tau)$. Here D is the noise strength and τ is the noise correlation time. This thesis is devoted to the study of damped stochastic system driven by colored noise governed by the above mentioned SDE for the stationary probability density function of x.

In this thesis we propose an analytical solution for the problem of damped stochastic system driven by colored noise. First we derive an effective Fokker-Planck equation (EFPE) for the above stochastic process $x(t)$ using the path integral formalism and by applying the steepest descent approximation technique. The stationary probability density function (SPDF) of the above stochastic process is derived analytically using the matrix continued fraction (MCF) method. We then

elaborate our theoretical procedure by taking up a case study on the damped bistable system driven by Gaussian Markovian noise. In order to compare our theoretical results, we carry out extensive digital simulation of the damped bistable system driven by Gaussian Markovian noise. In our digital simulation we compute the SPDF and the mean first passage time (MFPT) of the x process for the cases of white and colored noise. Our theoretical results for the SPDF of damped bistable system driven by colored noise agree very well with our corresponding digital simulation results thus validating the correctness of our theory. Further our digital simulation study is supposed to act as a benchmark for comparing future theoretical results in this area.

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