

A THESIS ON  
MECHANISMS OF TURBULENCE

BY

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CERTIFICATE

This is to certify that the thesis entitled 'MECHANISMS OF TURBULENCE' by Pradeep Kumar Sen has been prepared under my supervision in conformity with the rules and regulations of the Indian Institute of Technology, Delhi. I further certify that the thesis has attained a standard required for a Ph.D. degree of the Institute. The research report and results presented in the thesis have not been submitted for any degree in any other University.

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