

A THESIS ON
SOME PROBLEMS IN
BIVARIATE INTERPOLATION AND APPROXIMATION

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C E R T I F I C A T E

This is to certify that the thesis entitled "On Some Problems in Bivariate Interpolation and Approximation" which is being submitted by Mr. N. Jayarajan for the award of Doctor of Philosophy in Mathematics to the Indian Institute of Technology, New Delhi, is a record of bonafide research work. He has worked for the last three years under my guidance and supervision.

The thesis has reached the standard fulfilling the requirements of the regulations relating to the degree. The results obtained in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

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