

**ANALOG GRAVITY WITH SYNTHETICALLY  
SPIN-ORBIT-COUPLED  
BOSE-EINSTEIN CONDENSATE**

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**DEPARTMENT OF PHYSICS  
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BOSE-EINSTEIN CONDENSATE**

*by*

**INDERPREET KAUR**

**DEPARTMENT OF PHYSICS**

*Submitted*

**in fulfilment of the requirements of the degree of Doctor of Philosophy**

*to the*



**INDIAN INSTITUTE OF TECHNOLOGY DELHI**

**APRIL 2023**

*Under the inspiration of Almighty God, I dedicate  
this thesis to my loving Family!*

# CERTIFICATE

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This is to certify that the thesis titled, “**Analog Gravity with synthetically spin-orbit-coupled Bose-Einstein condensate**”, submitted by **Ms. Inderpreet Kaur**, to the Department of Physics, Indian Institute of Technology Delhi, for the award of the degree of **Doctor of Philosophy**, is a bona fide record of the research work carried out by her under my supervision and guidance. She has fulfilled the requirements for the submission of the thesis, which to the best of our knowledge has reached the required standard. The results obtained in this thesis have not been submitted in part or full to any other University or Institute for the award of any degree or diploma.

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**Inderpreet Kaur**

# Abstract

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Artificially induced spin-orbit coupling in ultracold atoms and the research avenue of analog gravity are excellent illustrations of implementing *Feynman's* envision of quantum simulators. We investigate the impact of the synthetic gauge fields in a Bose-Einstein condensate on realizing a quasi-two-dimensional sonic black hole. The velocity field in such spin-orbit-coupled Bose-Einstein condensate (BEC) breaks the irrotationality constraint, contrasting the usual superfluid BEC's case, and the system exhibits a non-zero value of angular momentum even without the existence of externally applied rotation. We study the condensate dynamics in a suitably generated laser-induced potential and investigate the effect of spin-orbit coupling on various properties of the resulting sonic black hole (SBH). The formation of a sonic black hole in an annular region, bounded by an inner and outer event horizon and elliptical ergosurfaces, is demonstrated.

We report amplifying density modulations due to the formation of such sonic horizons and illustrate how they alter the nature of analog Hawking radiation emitted from the sonic black hole. For identifying the emitted analog Hawking radiation, we compute the density-density correlation at different times using the truncated Wigner approximation accounting for quantum fluctuations in the simulation. However, implementation of the approximation requires the knowledge of the quasiparticle amplitudes in the trapped configuration, which we determine using the Bogoliubov-de Gennes formalism. The thermal nature of the emitted analog Hawking radiation is investigated using the computed correlation function. We firstly discuss the nature of the emitted analog Hawking radiation by considering radial densities alone for various angular directions. But, in a two-dimensional SBH, the pair of phonons can be spread over the whole quasi-two-dimensional plane exhibiting angular and radial correlations between them. Therefore, we examine the consequences of such correlations between the phonons spread over the entire plane on the emitted analog Hawking radiation.

In a later chapter, we also touch upon the issue of the spin-orbit coupled BEC experiencing counter-propagating laser beams with orthogonal polarization to

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understand the nature of the collective excitation spectrum in this configuration.

## शोध-सार

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अल्ट्राकोल्ड परमाणुओं में कृत्रिम रूप से प्रेरित स्पिन-ऑर्बिट युग्मन और एनालॉग ग्रेविटी के अनुसंधान एवेन्यू क्वांटम सिमुलेटर के *फेनमैन* की कल्पना को लागू करने के उत्कृष्ट उदाहरण हैं। हम अर्ध-द्वि-आयामी सोनिक ब्लैक होल को साकार करने पर बोस-आइंस्टीन कंडेनसेट में सिंथेटिक गेज फ़ील्ड के प्रभाव की जांच करते हैं। इस तरह के स्पिन-ऑर्बिट-युग्मित बोस-आइंस्टीन कंडेनसेट (बीईसी) में वेग क्षेत्र सामान्य सुपरफ्लूड बीईसी के मामले के विपरीत, इरोटेशनलिटी बाधा को तोड़ता है, और सिस्टम बाहरी रूप से लागू रोटेशन के अस्तित्व के बिना भी कोणीय गति का गैर-शून्य मान प्रदर्शित करता है। हम उचित रूप से उत्पन्न लेजर-प्रेरित क्षमता में घनीभूत गतिकी का अध्ययन करते हैं और परिणामी सोनिक ब्लैक होल (एसबीएच) के विभिन्न गुणों पर स्पिन-ऑर्बिट युग्मन के प्रभाव की जांच करते हैं। एक आंतरिक और बाहरी घटना क्षितिज और अण्डाकार अर्ग सतह से घिरे एक कुंडलाकार क्षेत्र में एक सोनिक ब्लैक होल का निर्माण प्रदर्शित किया गया है।

हम इस तरह के ध्वनि क्षितिज के गठन के कारण प्रवर्धित घनत्व मॉड्युलन की रिपोर्ट करते हैं और बताते हैं कि वे ध्वनि ब्लैक होल से निकलने वाले एनालॉग हॉकिंग विकिरण की प्रकृति को कैसे बदलते हैं। उत्सर्जित एनालॉग हॉकिंग विकिरण की पहचान करने के लिए, हम सिमुलेशन में क्वांटम उतार-चढ़ाव के लिए अलग-अलग समय पर घनत्व-घनत्व सहसंबंध की गणना करते हैं। हालाँकि, सन्निकटन के कार्यान्वयन के लिए फंसे हुए विन्यास में क्वासिपार्टिकल एम्पलीट्यूड के ज्ञान की आवश्यकता होती है, जिसे हम बोगोलीबॉव-डी गेनेस औपचारिकता का उपयोग करके निर्धारित करते हैं। उत्सर्जित एनालॉग हॉकिंग विकिरण की ऊष्मीय प्रकृति की गणना गणना सहसंबंध समारोह का उपयोग करके की जाती है। हम पहले विभिन्न कोणीय दिशाओं के लिए रेडियल घनत्व पर विचार करके उत्सर्जित एनालॉग हॉकिंग विकिरण की प्रकृति पर चर्चा करते हैं। लेकिन, एक द्वि-आयामी एसबीएच में फोनन की जोड़ी पूरे अर्ध-द्वि-आयामी विमान में फैल सकती है जो उनके बीच कोणीय और रेडियल सहसंबंध प्रदर्शित करती है। इसलिए, हम उत्सर्जित एनालॉग हॉकिंग विकिरण पर पूरे विमान में फैले फोनन के बीच ऐसे सहसंबंधों के परिणामों की जांच करते हैं।

बाद के एक अध्याय में, हम इस कॉन्फिगरेशन में सामूहिक उत्तेजना स्पेक्ट्रम की प्रकृति को समझने के लिए ऑर्थोगोनल ध्रुवीकरण के साथ स्पिन-ऑर्बिट युग्मित बीईसी का अनुभव करने वाले काउंटर-प्रॉपेगेटिंग लेजर बीम के मुद्दे पर भी स्पर्श करते हैं।

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