

# **INVESTIGATIONS ON DIFFRACTION IMAGES IN A POLARIZING MICROSCOPE WITH CROSSED POLARIZERS**

By

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Thesis submitted in fulfilment of the  
requirements of the degree of  
**DOCTOR OF PHILOSOPHY**

to the

**Indian Institute of Technology, Delhi**  
December, 1981

## ACKNOWLEDGEMENTS

I wish to express my deep sense of gratitude to Prof. A.K. Ghatak and Shri K. Singh, Assistant Professor for their valuable guidance, constant encouragement and critical discussions.

I wish to thank my colleagues Dr. A.K. Gupta, Miss Meenakshi, and Messers Braham Prakash, Rajesh Rampal and P.K. Gupta for their help and cooperation. The help given by the staff of the Computer Centre is acknowledged with pleasure. Thanks are also due to Dr. Bhanwar Singh, Dr. R.S. Daryan, Dr. V.N. Yadav, Dr. A.K. Sharma and Mr. Ramesh Duggel for helping me in many ways during the different stages of the preparation of this thesis.

My special thanks are also due to Mr. Vinod Bhargava, Managing Director, Harvin Optical and Glass Industries, Hyderabad for encouragement and providing leave for the completion of this work.

I am deeply indebted to my parents and other family members for their constant inspiration and moral support.

Finally, I wish to thank Miss Neelam Dhody for her efficient typing of this thesis.

  
( JAGPAL SINGH )

## ABSTRACT

The evaluation of the performance of an optical imaging system continues to be an important field of research and development because a perfect system can not be realized in practice. Imperfections due to geometrical aberrations, surface inaccuracy, the degree of coherence of light used, image motion etc. may considerably modify the performance of the optical system. When an optical imaging system has been designed and fabricated, it has to undergo many qualitative and quantitative tests to ascertain the extent to which it meets the aspirations of the designer. The use of Fourier techniques have led to optical system being analysed with the help of optical transfer function commonly known as the OTF. Since then, detailed investigations have been carried out concerning the imaging by optical system in the presence of intentional and unintentional modification of the pupil function. An interesting case of unusual type of pupil function is that of a polarizing microscope.

A polarizing microscope is one of the important tool in a variety of applications in different fields of analysis, especially in the studies of crystals. It is used to detect birefringence, location of optic axis of birefringence and the shape and orientation of index ellipsoid. Basically, it's a compound microscope equipped with a polarizer and an analyzer

but various other components are often incorporated. The basic theory of image formation of an aberration free polarizing microscope has been discussed in detail in many books (Clark, 1961; Slayter, 1970; Gifkins, 1970; McCrone, 1978). The diffraction theory of image formation of a polarizing microscope was initiated by Kubota, Inoue and Saito in two important papers (Kubota and Inoue, 1959; Kubota and Saito, 1960). They have shown that the diffraction image of a point source is intrinsically different from the conventional Airy disc of an aberration free system. Therefore, the imaging characteristics of this system cannot be investigated by applying the customary definition of resolution by Rayleigh or Strehl. Keeping in mind the complexity and the utility of the polarizing microscope, the present thesis extends the investigations on diffraction images of a polarizing microscope with crossed polarizers under realistic conditions of illumination and aberrations. Results are likely to have applications in setting of tolerances on the amount of aberration etc., for this type of unusual system.

The thesis is divided in seven chapters. In the first chapter an introduction to the subject of image formation is given. A brief summary of other types of microscope and characterization of aberration function has also been presented.

In studying the performance of microscopic optical system, the compact details of isolated objects are often required. Therefore, the diffraction images of various objects like degraded edge, bar, disk and annulus formed by a polarizing microscope with crossed polarizers have been investigated. The results have been presented graphically for different azimuths in Chapter II and III. The images are of unusual shape and strongly dependent upon the size of the object/coherence parameters.

The influence of rotationally symmetric third-order aberrations (defocusing and spherical aberration) on the performance of system is analysed in Chapter IV by calculating the point spread function and optical transfer function for various values of aberration coefficients. As it is impossible to reduce the aberrations of an optical system precisely to zero, the results for an optimally balanced fifth-order spherical aberration are also presented. The studies have been further extended for the case of third order off-axis aberrations and fifth-order optimally balanced coma. The results are shown in Chapter V. It has been observed that the point spread function and optical transfer function of the polarizing microscope are quite sensitive to small changes in the aberration coefficient. As the function occurring in integrals are oscillatory in nature, a great care has been taken in the computation and has been discussed in Chapter IV. The results

for the effect of sinusoidal phase errors arising due to the surface imperfections have been presented for the point spread function and optical transfer function in Chapter VI.

It is also known that the light sources and detector used, in general, exhibit significant variations over the spectrum. Therefore, the effect of polychromaticity of light source and detector response must be taken into account. The inclusion of the effect of polychromaticity on the OTF is found to be very complex in nature. Therefore, in Chapter VII, the modulation transfer function of diffraction limited polarizing microscope in polychromatic light has been evaluated for a number of common light sources and detectors so that the detector output can be predicted. The consideration of polychromatic MTF are of dual importance. Firstly, it represents a situation encountered frequently and secondly, it constitutes an upper limit on the performance obtainable with any optical system.

As the sources are never completely temporally coherent, we have also analysed the effect of temporal coherence on the point spread function of the system. It is shown that this effect is very small when compared with the effect of spatial coherence.

The work reported in the present thesis has resulted in the following publications:

1. Images of degraded edge objects formed by a polarizing microscope with crossed polarizers. Ind. J. Pure and Appl. Phys. (In Press).
2. Incoherent bar images formed by a polarizing microscope with crossed polarizers, Microscope. Acta. 78(1976) 149.
3. Imaging of incoherent disk objects by polarizing microscope with crossed polarizers, Opt. Acta. 23 (1976) 161.
4. Images of an annulus object in a polarizing microscope with crossed polarizers, Ind. J. Pure and Appl. Phys. 13 (1975) 840.
5. Effect of third-order aberrations on the point spread function of a polarizing microscope with crossed polarizers (submitted for publication).
6. Optical transfer function of a polarizing microscope with crossed polarizers afflicted with rotationally symmetric third-order aberrations (submitted for publication).
7. Optical transfer function of a polarizing microscope with crossed polarizers: Effect of off-axis aberrations, Optik (In Press).
8. Influence of Optimum balanced spherical aberration and coma on the performance of a polarizing microscope with crossed polarizers (submitted for publication).
9. Effect of low frequency sinusoidal phase errors on the performance of a polarizing microscope with crossed polarizers (submitted for publication).
10. Polychromatic optical transfer function of a polarizing microscope with crossed polarizers. Microscope Acta (In Press).
11. Effect of temporal coherence on the point spread function of a polarizing microscope with crossed polarizers. Microscope Acta 85 (1981) No. 1. Sept.

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