A NOVEL REPRESENTATION FOR ARMA PROCESSES : THEORY AND APPLICATIONS

by

SHIV DUTT JOSHI

Thesis submitted
in fulfilment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

DEPARTMENT OF ELECTRICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY, DELHI

1987

CERTIFICATE

This is to certify that the Ph.D. dissertation entitled, "A Novel Representation for ARMA Processes: Theory and Applications" presented by SHIV-DUTT-JOSHI, is a record of the bonafide research work he has done, during the period from July 1983 to October 1987 under my supervision. The results obtained in this dissertation have not been submitted for the award of any degree or diploma so far.

Surendra Prasad

Professor

Department of Electrical Engineering

Indian Insttute of Technology, Delhi

New Delhi-110016

TO THE WORD "LOVE"

A C K N O W L E D G E M E N T

No amount of words would suffice to communicate my deep sense of gratitude to my guide and supervisor, Prof.Surendra Prasad, for the invaluable guidance rendered to me during the course of this work. More than a guide, he was like a sculptor carving one of his masterpieces. It is he, who shaped my personality into what it is now.

I would like to specially thank Prof.S.C.Dutta Roy who not only looked after me as one of his own students during my guide's absence for a year but is also "the person" who motivated me to take up research.

It is with pleasure that I acknowledge my thanks to Mr.S.N.Gupta, SSO II, CARE, Lt.Cdr.Rajgopal, CARE and Miss Bindu Chandana, SSO II, CARE for the kind help provided to me during the preparation of this thesis and otherwise.

My esteemed thanks are also due to my mother whose sacrifices made me what I am today.

Lastly, but not the least, I would like to thank the person nearest to my heart, my wife Archana, for providing me much needed love and inspiration throughout the course of this work.

Shiv-Dutt

(Shiv-Dutt-Joshi)

ABSTRACT

In applications concerning time-series analysis and stochastic signal processing, one of the fundamental issues of interest is that of modeling a given stochastic sequence. This modeling essentially requires finite parameterization of the infinite autocorrelation sequence of the given time series. The autoregressive moving average (ARMA) models are an important class of such parsimonious approximants. In the literature, which is very rich, various techniques exist for the identification of ARMA models from a given stochastic sequence. Most of these techniques, however, suffer from certain drawbacks which either make them infeasible in a practical situation (due to the high computational complexity involved) or render the resulting solution suboptimal (due to the nonlinearity of the problem).

In the present work, an attempt has been made to explore in depth the structure of ARMA processes starting from first principles. By regarding the random variables as vectors in a Hilbert space, some interesting and useful results have been obtained concerning the nature of the multiple step ahead predictors of a time series. It is demonstrated that the predictor results obtained here, when used in conjunction with the finite dimensional structure of the predictor space associated with these processes, lead to an entirely new representation for them. This representation, termed here as the "predictor space representation" or PSR, and obtained by exploiting some geometrical features of these processes, has a

number of useful and interesting properties. One of the major applications of this representation is in the ARMA parameter estimation problem. Since the parameter space associated with the ARMA problem is not a linear space, an order recursive solution is usually not possible directly. With the help of the new PSR, however, this problem can be decoupled into the estimation of two sets of parameters whereby each of them has a linear parameter space and therefore can be recursively updated.

With the new PSR at our disposal, we seek a time as well as order recursive solution to the ARMA filtering and parameter estimation problems. We present a Hilbert space formalism for the ARMA modeling problem from a given set of raw data. We, then, derive a new projection operator update formula which is particularly suited for the recursive solution of the underlying problem. With the help of this projection operator update formula and the Hilbert space formalism for the given data case, we develop computationally efficient, exact, recursive least squares algorithm for the ARMA filtering problem. These recursions, we observe, can be put in the form of a filter structure.

The ARMA parameters, however, are not directly available from the coefficients of the aforementioned structure. For this purpose, two update relations are derived and used to obtain recursive, exact least squares algorithm for AR parameter estimation from the coefficients of the structure. Whereas, an efficient algorithm for the computation of the AR parameters could then be developed, the derivation of similar efficient schemes for MA parameter estimation has been somewhat illusive.

We, however, present two algorithms for MA parameter estimation, one being exact (in the least squares sense) but computationally inefficient, and the other approximate but efficient. Some numerical results are also presented to demonstrate the feasibility of the proposed algorithms.

We also develop here a generalized form of the predictor space representation, for expressing the input-output relationship of an arbitrary stochastic system in the new framework, without constraining the input to be white. This helps us to develop a new insight into the stochastic system realization problem with arbitrary inputs.

Finally, we also attempt here to present a unified view of some related theories and results which have been developed in other areas, such as design of digital filters using second order statistics, the stochastic system realization problem etc. It is demonstrated that the new representation theory introduced in this dissertation provides some remarkably new interpretations of several such results, sometimes even providing a physical picture where it was not apparent before.

LIST OF CONTENTS

PAGE

CHAPTER 1

Introduction

CHAPTER 2

A Novel Predictor Space Representation for ARMA Processes

CHAPTER 3

Exact Recursive Least Squares Algorithm for ARMA Filtering

CHAPTER 4

ARMA Parameter Estimation Algorithms

CHAPTER 5

Predictor Space Representation: Generalization and Applications