# MULTILEVEL ADAPTIVE WAVELET METHODS FOR SOLUTION OF PDEs


by

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*to the*


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*Dedicated to*

*My Family*

# Certificate

This is to certify that the thesis entitled **"Multilevel Adaptive Wavelet Methods for Solution of PDEs"** submitted by **"Mr. Ratikanta Behera"** to the Indian Institute of Technology (IIT) Delhi, for the award of the Degree of Doctor of Philosophy, is a record of the original bonafide research work carried out by him under my supervision and guidance. The thesis has reached the standards fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted in part or full to any other university or institute for the award of any degree or diploma.

New Delhi

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# Abstract

Mathematical modeling of problems in science and engineering typically involves solving partial differential equations (PDEs). The ability of wavelet based numerical methods to resolve localized structures appearing in the solution without drastic increase in the number of the unknowns, enables us to pursue a different avenue of research. Because most of the problems with localized structures or sharp transitions might occur intermittently anywhere in the computational domain or might change their locations and scales in space and time. Multilevel adaptive wavelet methods are ideally suited for PDEs encountered in many applications, such as in fluid dynamics, materials science, inverse problems, computer graphics, and pattern formation etc.

The main objective of this thesis is to develop multilevel adaptive wavelet methods for solutions of PDEs in one- and two-dimensions, and on the sphere. Solving one- and two-dimensional PDEs are based on Daubechies wavelet on an arbitrary grid, and that of the PDEs on the sphere is based on spherical wavelet over spherical geodesic grid.

When solving PDEs on the sphere, it is necessary to approximate fundamental differential operators like gradient, divergence, curl, and Jacobian operators. Therefore, efficient adaptive tools are needed to approximate these differential operators. To this and, a hierarchical finite difference scheme based on the wavelet multilevel decomposition is used to approximate these differential operators. The multilevel structure provides a simple way to adapt the computation to the local structure of the differential operators so that the high resolution computations are performed only in regions where singularities or sharp transitions occur. The accuracy and efficiency of the new approximation technique is verified by numerical experiments. In contrast with other approximation schemes, this approach can be extended easily to other curved manifolds by choosing an appropriate coarse approximation and using recursive surface subdivision.

Application of multilevel adaptive wavelet methods are addressed for PDEs in one- and two-dimension, and on the sphere. In one-dimension, we have solved modified Camassa-Holm (mCH) equation, modified Degasperis-Procesi (mDP) equation and Schrodinger equation (one-dimension and two-dimension). We have given a brief review of the one-dimensional setting of multilevel adaptive wavelet method and extended this method to two-dimensional multilevel adaptive wavelet method. Furthermore, we have discussed spherical Schrodinger equation and linear fourth order diffusion equation over adaptive optimal spherical geodesic grid. In addition, we have also solved convection dominated problem over spherical geodesic grid. The results indicate that the computational grid and associated wavelets are very efficiently adapted to the local irregularities of the solution in order to resolve sharp transition regions that clearly demonstrate the effectiveness of the multilevel adaptive wavelet method.

Multilevel adaptive wavelet method has been used to solve non-divergent barotropic vorticity equation (Rossby-Haurwitz wave and Verkley's dipole modons) over spherical geodesic grid. Here it has depicted that mass and potential enstrophy of these numerical solutions are conserved with high degree of precision. Advection problems and the shallow water equations are the set of equations used to model many fluid flows. They are particularly well suited and often used to test numerical methods for weather prediction. We have carried out the advection of the cosine bell (test case-1), divergent deformational flow [Nair and Lauritzen, J. Comput. Phys., 229 (2010) 8868–8887], and steady state zonal flow (test case-2) of [Williamson et al., J. Comput. Phys., 102 (1992) 211–224]. It has revealed that the numerical solutions of these problems were accurate, computationally efficient with capturing, identifying, and analyzing local structure of the problems with respective results.

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