MICROMECHANICAL MODELLING OF ELASTIC AND STRENGTH PROPERTIES OF SNOW FROM MICRO-TOMOGRAPHIC IMAGES

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MICROMECHANICAL MODELLING OF ELASTIC AND STRENGTH PROPERTIES OF SNOW FROM MICRO-TOMOGRAPHIC IMAGES

by

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Dedicated to my beloved parents and family

CERTIFICATE

This is to certify that the thesis entitled **"MICROMECHANICAL MODELLING OF ELASTIC AND STRENGTH PROPERTIES OF SNOW FROM MICRO-TOMOGRAPHIC IMAGES"** being submitted by **Mr. Praveen Kumar Srivastava** is the report of bonafide research work carried by him under my supervision. This thesis has been prepared in conformity with the rules and regulations of the INDIAN INSTITUTE OF TECHNOLOGY DELHI. I further certify that the thesis has attained a standard required for the award of **DOCTOR OF PHILOSOPHY** degree of the institute. The research reported and the results presented in the thesis have not been submitted, in part or full to any other institute or university for the award of any other degree or diploma.

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ABSTRACT

Snow is a porous material consisting of a complex interconnected network of sintered ice crystals. The underlying variations in ice volume fraction and microstructural fabric lead to large heterogeneity in macroscale mechanical properties and failure behaviour of snow. The present study develops a tomographic-image based approach to define the macroscale mechanical behaviour of snow using multiscale computational homogenization methods. Assuming linear elastic properties of ice and periodic boundary conditions, micro-finite element (μFE) models built directly from the X-ray micro-computed tomography (μCT) images were used to compute the effective orthotropic stiffness and compliance tensors for a wide range of snow densities and morphologies. The effective isotropic Young's moduli and Poisson ratios derived from these elasticity tensors compare quite well with previously published results thereby validating the numerical modelling approach adopted. The μCT image dataset also allowed to determine the degree of anisotropy and principle planes of orthotropic symmetry of microstructure by means of second-rank surface- and volume-based fabric tensors. The isotropic elasticity model based on ice volume fraction could explain 89% of the variability of the stiffness tensor computed by the μFE model with mean relative norm error of 43%. In contrast, the orthotropic elasticity model based on a second-rank fabric tensor and the volume fraction raised the adjusted coefficient of determination (r_{adj}^2) to 97% with mean relative norm error of 28%.

Motivated by successful estimates of elastic properties, the study was extended to nonlinear μFE analysis for modelling of anisotropic strength properties and multiaxial failure criterion. A damage based elasto-plastic constitutive law with Drucker-Prager yield criteria is

used for ice to account for the tension-compression asymmetry in macroscopic strength behaviour of snow. The nonlinear μFE models were subjected to 45-48 different load cases each up to failure. The simulations captured the macroscale tension-compression strength asymmetry as well as the anticipated effect of density and anisotropic fabric on the uniaxial strength behavior of snow. While the anisotropic failure strengths were found to be strongly correlated with corresponding elastic moduli, the failure strains were found to be independent of moduli and density. The μFE derived uniaxial and biaxial strength data were fitted with a Tsai-Wu type orthotropic failure surface in stress space and a strong relationship is demonstrated between volume fraction, fabric and multiaxial strength behaviour of snow. A minimal set of four constants and two exponents is identified for the description of multiaxial failure surface as a function of ice volume fraction and fabric. Neglecting fabric information and using an isotropic model based on only ice volume fraction yielded r^2_{adj} of 0.89 with associated model norm error of 40.8%. Depending on fabric measure used, the fabric-based orthotropic strength model provided r^2_{adj} of 0.95 and norm error in the range of 27-28%. The predictions of fabric based strength models are not only consistent with experimental data from the literature but also extend the current knowledge of failure behaviour to multiaxial load cases that can hardly be realized in a laboratory experiment.

संछिप्त विवरण

हिम एक छिद्रिल पदार्थ है जो कि बर्फ के निसदित क्रिस्टल्स द्वारा बना एक मिश्रित और जटिल जालतन्त्र है। बर्फ के अंशखंड और सूछ्म संरचना फैब्रिक में अन्तर्निहित बदलाव, हिम के स्थूल यांत्रिक गुणों तथा फेलियर व्यवहार में बड़ी विषमता का मूल कारण बनते हैं। वर्तमान अध्ययन हिम के मैक्रो-स्केल यांत्रिक व्यवहार को परिभाषित करने के लिए एक टोमोग्राफिक-छवि आधारित दृष्टिकोण विकसित करता है जिसमे मल्टिस्केल कम्प्यूटेशनल होमोजेनाइजेशन विधियों का उपयोग किया गया है। बर्फ के सूक्ष्म प्रत्यास्थ गुणों और पीरियाडिक बाउंड्री कंडीशंस को मानते हुए, हिम के सूक्ष्म-परिमित एलिमेंट (μFE) मॉडल का निर्माण एक्स-रे माइक्रो-कंप्यूटेड टोमोग्राफी (μCT) छवियों से सीधे किया गया है और इनका उपयोग एक विस्तृत हिम घनत्व और आकारिकी के सैंपल श्रृंखला की प्रभावी ऑर्थोट्रोपिक प्रत्यास्थता और कंप्लायंस टेंसरों की गणना के लिए किया गया। इन प्रत्यास्थता टेंसर्स से प्राप्त प्रभावी आइसोट्रोपिक यंग'स मोडुली और पॉइसन अनुपात पहले के साथ प्रकाशित परीणामो के साथ काफी अच्छी तरह से मेल करते हैं, जिससे वर्तमान अध्य्यन मे उपयोग किये गये संख्यात्मक मॉडलिंग दृष्टिकोण को मान्यता मिल्ती है। द्वितय रैंक के सतह- और आयतन-आधारित फेब्रिक टेंसर्स, μCT छवि डेटासेट के स्ट्रक्चरल एनिसोट्रॉपी का परिमाण और माइक्रोस्ट्रक्चर के प्रिंसिपल प्लेन्स ऑफ़ ऑर्थोत्रोपिक सिमिट्री का निर्धारण भी सुनिश्चित करते है। 43% की माध्य सापेक्ष त्रुटिमान के साथ, बर्फ अंशखंड पर आधारित आइसोट्रोपिक प्रत्यास्थ मॉडल, μFE मॉडल द्वारा आंकलित प्रत्यास्थता की परिवर्तनशीलता की 89% तक व्याख्या कर सकता है। इसके विपरीत, बर्फ अंशखंड और दूसरी रैंक के फैब्रिक् टेंसर पर आधारित ऑर्थोट्रोपिक प्रत्यास्थ मॉडल ने 28% की औसत सापेक्ष त्रुटि के साथ समायोजित गुणांक (r^2_{adj}) को 97% तक बढ़ा दिया।

प्रत्यास्थ गुणों के सफल अनुमानों से प्रेरित होकर, अध्ययन को नोनलीनिअर μFE विश्लेषण से एनीसोट्रोपिक सामर्थ्य गुणों और मल्टीएक्सिअल फेलियर क्राइटेरिया की

मॉडलिंग के लिए आगे बढ़ाया गया। हिम के मैक्रोस्कोपिक टेंशन-कम्प्रेशन असममितता को संज्ञान में लेते हुए, बर्फ के लिए ड्रकर-प्रेजर यील्ड मानदंड के साथ एक डैमेज आधारित इलास्टो-प्लास्टिक कोंस्टीटूटिव नियम का उपयोग किया गया। प्रत्येक नोनलिनिअर µFE मॉडल का अध्यन 45-48 विभिन्न लोड केसेस के लिए फेलिअर तक किया गया। सिम्युलेशन्स ने मैक्रोस्कोपिक टेंशन-कम्प्रेशन असममितता के साथ-साथ सामर्थ्य गुणों पर हिम घनत्व के प्रत्याशित प्रभाव को भलिभांति पकडा। जहाँ एक तरफ अनिसोट्रोपिक फेलिअर स्ट्रेंथ संबंधित प्रत्यास्थ मोडुली के साथ दृढ़ता से सहसंबद्ध पाई गई, फेलिअर स्ट्रेन, प्रत्यास्थ मोडुली और घनत्व से स्वतंत्र पाये गये। µFE मॉडल से व्युत्पन्न एकाक्षीय और द्विअक्षीय स्ट्रेंथ डेटा को स्ट्रेस स्पेस में Tsai-Wu टाइप ओर्थोट्रोपिक फेलिअर सतह के साथ फिट किय गया और बर्फ के आयतन अंश, फेब्रिक और बहुअक्षीयस्ट्रेंथ व्यवहार के बीच मजबूत सम्बंध प्रदर्शित किया गया। बर्फ की मात्रा-अंश और फेब्रिक के रूप में मल्टीएक्सिअल फेलिअर सतह के वर्णन के लिए चार स्थिरांक और दो घातांक का न्यूनतम सेट पहचाना गया। फेब्रिक की जानकारी की उपेक्षा करते हुए और केवल आइस वॉल्यूम अंश के आधार पर एक आइसोट्रोपिक मॉडल का उपयोग करके r^2_{adj} का मान 0.89 प्राप्त हुआ जिसमे संबंधित मॉडल मानक त्रुटि 40.8% थी। उपयोग किए गए फेब्रिक के माप के आधार पर, फेब्रिक-आधारित ऑर्थोट्रोपिक स्ट्रेंथ मॉडल से r^2_{adj} का मान 0.95 प्राप्त हुआ और इसमे मानक त्रुटि की सीमा 27-28% थी। फेब्रिक आधारित स्ट्रेंथ मॉडल की भविष्यवाणियां न केवल प्रयोगात्मक डेटा (जोकि लिट्रेचर् से लिया गया है) के अनुरूप हैं बल्कि फेलिअर व्यवहार के मौजूदा ज्ञान को भी बहुअक्षीय लोड अवस्थाओ तक विस्तारित करते हैं जिन्हें प्रयोगशाला प्रयोग में शायद ही रियलाइज्ड किया जा सकता है।

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T	Temperature
T_h	Homologous temperature
T_{melt}	Melting point temperature
TG, ∇T	Temperature gradient
PP	Fresh snow crystals
RG	Rounded grains
FC	Faceted crystals
DH	Depth hoar crystals
DF	Decomposing and fragmented precipitation particles
$\boldsymbol{\varepsilon}$	Strain tensor
$\dot{\varepsilon}$	Strain rate
μCT	X-ray micro-computed tomography
ρ_s	Density of snow
ρ_{ice}	Density of ice
E	Young's modulus
E_{ice}	Young's modulus of ice
μ_{ice}	Poisson's ratio of ice
σ_t	Tensile strength
$\sigma_{ice,t}$	Tensile strength of ice
$\sigma_{ice,c}$	Compressive strength of ice
τ^I, τ^{II}, τ^Y	Shear strength (superscripts corresponds to grouped classes or author)
τ	Shear stress
c	Cohesion
ϕ	Internal friction angle
σ_n	Normal stress
p_c	Confining pressure
MC	Mohr-Coulomb failure criterion
MCC	Mohr-Coulomb-Cap model

DEM	Discrete element model
μFE	Micro-finite element
RVE	Representative Volume Element
MIL	Mean Intercept Length
SLD	Star Length Distribution
SVD	Star Volume Distribution
υ_P	Porosity
υ_s	Ice volume fraction
$\Delta_i(x)$	Local ice thickness at a point x
$P(\Delta_i)$	Ice thickness distribution
$P(\Delta_p)$	Pore thickness distribution
h_{ice}	Mean ice thickness
h_{pore}	Mean pore thickness
$\boldsymbol{\omega}$	Orientation unit vector
$N_I(\boldsymbol{\omega})$	Number of intercept segments for direction $\boldsymbol{\omega}$
$\boldsymbol{\omega}_i$	Orientation unit vector along i^{th} direction
(x_i, y_i, z_i)	Direction cosines along i^{th} direction
$L_i(\boldsymbol{\omega})$	Length of i^{th} intercept segment
$\bar{L}(\boldsymbol{\omega})$	Mean intercept length for direction $\boldsymbol{\omega}$
$\bar{L}(\boldsymbol{\omega}_i)$	Mean intercept length distribution for n directions $\boldsymbol{\omega}_i$
$\mathbf{X}$	Material tensor
$\mathbf{H}$	MIL fabric tensor
$\overline{\upsilon_V^*}$	Star volume component along direction $\boldsymbol{\omega}$
$s(\boldsymbol{\omega})$	Star length component along direction $\boldsymbol{\omega}$
SLD	Star length distribution fabric tensor
SVD	Star volume distribution fabric tensor
$\mathbf{M}$	Second rank fabric tensor
m_i: $(m_1,\ m_2,\ m_3)$	Normalized eigenvalues of fabric tensor
$\mathbf{m}_i$	Eigenvectors of fabric tensor
$tr(\mathbf{M})$	Trace of fabric tensor

I	Isotropy index
EI	Elongation index
l_i	Length scales
l_{micro}	Microscale
l_{meso}	Mesoscale
l_{RVE}	RVE length scale
l_{RVE,υ_s}	RVE size w.r.t. ice volume fraction
l_{macro}	Macroscale
V	Domain of RVE
∂V	Boundary of RVE
δ	Scale decoupling ratio i.e. $\delta := l_{RVE}/l_{micro} \gg 1$
L	Edge length of maximum available cubic image volume
d	Edge length of cubic sub-volumes
CV	Coefficient of variation
$(\upsilon_s)_{CV}$	CV of ice volume fraction
α_0, β	Constant and exponent of power law function
δ_{υ_s}	Dimensionless ratio i.e. number of heterogeneities/grains in RVE
δ_M	Dimensionless ratio $\delta_M := L/h_{ice}$
$\delta_M{}'$	Dimensionless ratio $\delta_M := L/h_{pore}$
$\mathbf{C}_{ijkl}$, $\mathbf{C}$	Stiffness tensor
$\mathbf{C}_{ijkl}^{eff}$, $\mathbf{C}^{eff}$	Effective stiffness tensor
$\mathbf{S}_{ijkl}$, $\mathbf{S}$	Compliance tensor
$\boldsymbol{\sigma}_{ij}$, $\boldsymbol{\sigma}$	Stress tensor components
$\bar{\boldsymbol{\sigma}}$	Uniform stress tensor averaged over a small material volume
$\boldsymbol{\varepsilon}_{kl}$	Strain tensor components at a material point
$\bar{\boldsymbol{\varepsilon}}$	Uniform strain tensor averaged over a small material volume
$\mathbf{C}_{ijkl}^{app}$	Apparent stiffness tensor
$\mathbf{C}_{SUBC}^{app}$	Apparent stiffness tensor under SUBC
$\mathbf{C}_{KUBC}^{app}$	Apparent stiffness tensor under KUBC

$\mathbf{C}_{\mathrm{PBC}}^{\mathrm{app}}$	Apparent stiffness tensor under PBC
KUBC	Kinematic uniform boundary conditions
SUBC	Static uniform boundary conditions
PBC	Periodic boundary conditions
x	a point in macroscale continuum
y	a point in RVE domain V associated with x
$\boldsymbol{\eta}_{\mathrm{i}}$	Virtual displacement field
$\mathbf{t}_{\mathrm{i}}$	Traction vector
$\partial \mathrm{V}_{\mathrm{t}}$	Part of boundary ∂V where the external traction vector is prescribed
ξ	Space of virtual displacement
$\mathbf{u}_{\mathrm{i}}$	Displacement fields
$\mathbf{C}^{\mathrm{ice}}$	Stiffness tensor of ice
$\mathbf{E}_{\mathrm{ij}}, \mathbf{E}$	Cartesian components of macroscopic strain tensor
$\langle \boldsymbol{\varepsilon}_{\mathrm{ij}} \rangle$	Mean strain tensor over V
$\boldsymbol{\Sigma}_{\mathbf{ij}}$	Cartesian components of macroscopic stress tensor
$\langle \boldsymbol{\sigma}_{\mathrm{ij}} \rangle$	Mean stress tensor over V
$\mathbf{n}$	a vector normal to ∂V
$\mathbf{v}_{\mathrm{i}}$	Fluctuations in displacement field $\mathbf{u}_{\mathrm{i}}$
$\langle \quad \rangle$	Volumetric averaging operator
e_{ij}	Unit macroscopic strains
$\mathrm{C}_{\mathrm{FE}_{\mathrm{aniso}}}$	μFE derived anisotropic stiffness tensor
$\mathrm{c}_{\mathrm{ij}}, \delta_{\mathrm{ij}}$	Components of Stiffness matrix
Obj	Orthotropy objective function
$\mathbf{C}_{\mathrm{FE}_{\mathrm{ortho}}}$	μFE derived orthotropic stiffness tensor
$\mathrm{NE}^{\mathrm{ortho}}$	Relative norm error by forcing orthotropy
E_{i}, G_{ij} and ν_{ij}	Orthotropic engineering constants
$\mathrm{E}_{\mathrm{eff}}$	Effective isotropic Young's modulus
ν_{eff}	Effective isotropic Poisson's ratio
$\mathrm{k}_{\mathrm{eff}}$	Effective isotropic Bulk modulus
$\mathrm{G}_{\mathrm{eff}}$	Effective isotropic Shear modulus

k_R	Reuss bound on Bulk modulus
k_V	Voigt bound on Bulk modulus
G_R	Reuss bound on Shear modulus
G_V	Voigt bound on Shear modulus
s_{11}	Components of compliance matrix
$\lVert \quad \rVert$	Norm operator
ISO	Isotropic elasticity model
$\{E_0, \nu_0, k\}$	ISO model constants in compliance form
$\{\lambda_0, G_0, k\}$	ISO model constants in stiffness form
$\mathbf{I}$	Second rank identity tensor
$\otimes$	Double tensorial product: $\mathbf{K} = \mathbf{A}\otimes\mathbf{B} \Longrightarrow K_{ijkl} = A_{ij}B_{kl}$
$\underline{\overline{\otimes}}$	Double tensorial product: $\mathbf{K} = \mathbf{A}\underline{\overline{\otimes}}\mathbf{B} \Longrightarrow K_{ijkl} = \frac{1}{2}\left(A_{ik}B_{jl} + A_{il}B_{jk}\right)$
$\mathcal{G}$	Symmetry group for general orthotropy
$\mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3$	Reflections with respect to orthogonal planes
$\mathbf{M_1}, \mathbf{M_2}, \mathbf{M_3}$	Structural tensors associated with $\mathcal{G}$
$\mathbf{Q}$	Second rank tensor representation of the orthogonal group $\mathcal{G}$
Ψ	Orthotropic free energy potential
ZC	Zysset-Curnier model
$\{E_0, \nu_0, G_0, k, l\}$	ZC model constant in compliance form
$\{\lambda_0, \lambda_0', G_0, k, l\}$	ZC model constant in stiffness form
r^2_{adj}	Adjusted coefficient of determination
NE^{model}	Model norm error
$f(\boldsymbol{\sigma})$	Scalar function of stress
$\mathbf{F}$, $\mathbb{F}$, $\mathbb{F}_{ijk}$	Second, fourth and sixth rank strength tensors
σ^-_{ii}, C_i	Uniaxial compressive strengths along the axis $\mathbf{e}_i$
σ^+_{ii}, T_i	Uniaxial tensile strengths along the axis $\mathbf{e}_i$
τ_{ij}, S_{ij}	Pure shear strength in the plane $\mathbf{e}_i$, $\mathbf{e}_j$
χ_{ij}	Normal stress interaction coefficients
$\{\sigma_0^{\pm}, \tau_0, \chi_0, p, q\}$	Fabric-based orthotropic strength model constants
K_I	Mode-I stress intensity factor

K_{Ic}	Critical mode-I stress intensity factor
ω	Scalar damage variable
S_D	Damaged area
S	Total area
$\bar{\sigma}$	Effective stress
E_0	Elastic modulus in undamaged state
E	Elastic modulus in damaged state
κ_d	Damage driving variable
g_d	Function of κ_d
$f_d(\varepsilon, \kappa_d)$	Damage loading function
$\mathbf{C_0}, \mathbf{C}$	Stiffness tensor in undamaged and damaged state
ε_{eq}	Equivalent strain
$\boldsymbol{\varepsilon_e}$	Elastic strain
$\boldsymbol{\varepsilon_p}$	Plastic strain
$F(\bar{\boldsymbol{\sigma}}, \sigma_Y)$	Scalar yield function
σ_Y	Yield stress
$\tilde{\boldsymbol{\sigma}}$	Equivalent stress
α_{DP}	Drucker-Prager model parameter
σ_{Y0}	Initial yield stress
H	Hardening modulus
$\bar{\varepsilon}_p$	Equivalent plastic strain
$\mathbf{s}, \mathbf{p}$	Deviatoric and hydrostatic component of effective stress
σ_C , σ_T	Uniaxial strength in compression and tension
$\dot{\lambda}$	Rate of the plastic multiplier
g	Function of effective stress $\bar{\boldsymbol{\sigma}}$
$\mathbf{N}$	Gradient of yield function
$\mathbf{Q}$	Gradient of plastic potential
$\boldsymbol{\delta_{ij}}$	Kronecker delta
$\sigma_{Y0_{max}}$	Maximum yield stress
$\bar{\varepsilon}_{pD}$	Critical equivalent plastic strain

G_f	Fracture energy release rate
L_c	Characteristic length associated with elements
$\bar{\varepsilon}_{pf}$	Equivalent plastic strain at failure
$\bar{\delta}_p$	Equivalent plastic displacement beyond yield
$\bar{\delta}_{pf}$	Equivalent plastic displacement at failure
K_{ice}	Fracture toughness of ice
k_p	Constant of exponential damage evolution model
γ	Regression constant of linear scaling relation
$RVE_{elastic}$	RVE for elastic properties
$RVE_{structure}$	RVE w.r.t. microstructure parameter
c	Propagation speed of transverse collapse
H	Slab thickness
ν	Slab Poisson's ratio
Δy	Total vertical collapse of weak layer
ρ	Mean slab density
g	Acceleration due to gravity
PST	Propagation saw test
E_{N1}, E_{N2}, E_{N3}	Normalized Young's moduli
$G_{N12}, G_{N13}, G_{N23}$	Normalized shear moduli
I_{Mech}	Mechanical isotropy
EI_{Mech}	Mechanical elongation
DV	Percentage damaged volume
D_ω	Mean damage of RVE
N_e	Number of elements in the µFE model
C_{N1}, C_{N2}, C_{N3}	Normalized compressive strengths
T_{N1}, T_{N2}, T_{N3}	Normalized tensile strengths
$S_{N12}, S_{N13}, S_{N23}$	Normalized shear strengths