

LEXI-SEARCH OPTIMIZATION IN ALLOCATION  
PROBLEMS

by

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*submitted*

*in fulfilment of the requirements of the degree of*

*DOCTOR OF PHILOSOPHY*

*to the*



INDIAN INSTITUTE OF TECHNOLOGY, DELHI

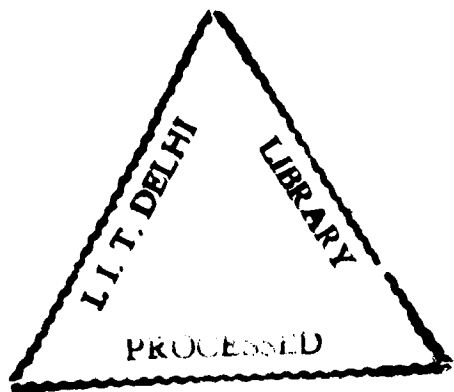
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## Certificate

This is to certify that the thesis entitled *LEXI-SEARCH OPTIMIZATION IN ALLOCATION PROBLEMS* which is being submitted by *Shalini Arora* for the award of the degree of *DOCTOR OF PHILOSOPHY* in *MATHEMATICS* to the *INDIAN INSTITUTE OF TECHNOLOGY, DELHI*, is a bonafide record of research work done under my guidance and supervision.

The thesis has reached the standard fulfilling the requirements of the regulations relating to the degree. The results obtained in the thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

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*to*

*my mother, brother*

*&*

*to all those I owe more than I can possibly express*

*with love and thanks*

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(SHALINI ARORA)

## ABSTRACT

*In the thesis entitled LEXI-SEARCH OPTIMIZATION IN ALLOCATION PROBLEMS an attempt is made to develop algorithms and solution methodologies for the various allocation problems with the help of lexi-search approach - an implicit enumeration technique, and to study some applications in various transportation problems.*

*The thesis consists of five chapters. Chapter 1 is an introductory one and contains a brief review of the related work and summary of the work presented in the thesis.*

*Chapter 2 and Chapter 3 are focussed on the applications of the lexi-search approach in obtaining optimal solutions of various allocation problems viz. time minimizing assignment problem(TMAP) and a few of its variants, bottleneck bulk transportation problem(BBTP). It is being assumed that all the facilities(sources) start working(supplying) simultaneously but a facility doing more than one job(source supplying to more than one destination) does so one after the other in any order. The objective function is the maximum of the total time taken by each facility(source) to complete all the jobs(serve all the destinations) assigned to it and is called the makespan. Aim is to minimize this makespan without violating the feasibility constraints which are different for various problems presented in the two chapters.*

*In Chapter 4, procedure is developed to find a lexicographic optimal solution(LOS) of lexicographic general optimization problem(LGOP) in context of different transportation problems viz. cost minimizing transportation problem(CMTP), time minimizing transportation problem(TMTP) and their variants, bottleneck bulk transportation problem(BBTP) and multi commodity transportation problem(MCTP). An (LGOP) is the one in which in addition to optimizing the highest priority objective one is also interested in optimizing the second highest priority objective and so on.*

*Chapter 5 studies two different aspects of (TMTP). The first one is a standard (TMTP) in which parallel transportation is done from all the sources but a source*

supplying to more than one destination does so one after the other in any order and again lexi-search approach is utilized to obtain an optimal solution. The second one is a phased (TMTP) in which total availability is more than the total requirement and the supply is done in two phases. In the first phase the minimum requirement of each destination is met and in the second phase the left over quantities are shipped to destinations. Parallel transportation is done in both the phases. An algorithm is developed to obtain the optimal schedules for both the phases by investigating lexicographic feasible solutions of a related standard (TMTP).

Computational results of some of the problems of first two chapters are reported in the Appendix. Numerical examples are given to illustrate the various algorithms. The bibliography is provided at the end of the thesis. The number(s) in the bracket [ ] refers to the articles/books mentioned in the bibliography. The results which are common for all the sections and the subsections of any chapter are reported in the beginning of that chapter and are numbered as ChA, ChB, ChC and so on where, Ch stands for the number of the chapter and A stands for the first general result of that chapter, B stands for the second general result of the chapter Ch and so on. Otherwise, numbering of various results(equations) is such that the first number represents the chapter, second the section, third the subsection(if it is there in a particular section) and third(fourth) the serial number of the result(equation) in that section(subsection). The problems are also numbered in the similar way except that there is P in the beginning.

## Glossary of abbreviations used in the thesis

TMAP	Time Minimizing Assignment Problem
BBTP	Bottleneck Bulk Transportation Problem
LOS	Lexicographic Optimal Solution
LGOP	Lexicographic General Optimization Problem
CMTP	Cost Minimizing Transportation Problem
TMTP	Time Minimizing Transportation Problem
BTP	Bulk Transportation Problem
GAP	Generalized Assignment Problem
MCTP	Multi – Commodity Transportation Problem
PTMTP	Phased Time Minimizing Transportation Problem
CMAP	Cost Minimizing Assignment Problem
APSC	Assignment Problem with Side Constraints
QAP	Quadratic Assignment Problem
ITMAP	Imbalanced Time Minimizing Assignment Problem
CMBTP	Cost Minimizing Bulk Transportation Problem
LCMTP	Lexicographic Cost Minimizing Transportation Problem
LTMTP	Lexicographic Time Minimizing Transportation Problem
LBBTP	Lexicographic Bottleneck Bulk Transportation Problem
LMTP	Lexicographic Multicommodity Transportation Problem
STMTP	Standard Time Minimizing Transportation Problem

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