MODELING OF SLENDER PIEZOELECTRIC ENERGY HARVESTER BY VARIATIONAL ASYMPTOTIC METHOD

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by

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in fulfilment of the requirements for the award of the degree of Doctor of Philosophy

to the

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Dedicated to

My Teachers

My Parents

My Husband

CERTIFICATE

This is to certify that the thesis entitled **"MODELING OF SLENDER PIEZOELECTRIC ENERGY HARVESTER BY VARIATIONAL ASYMPTOTIC METHOD"** being submitted by **Mrs. Shreya Banerjee** to Indian Institute of Technology Delhi for the award of degree of **Doctor of Philosophy** in Applied Mechanics is a record of original bonafide research work carried out by her under my supervision and guidance. The thesis work in my opinion has reached the requisite standard fulfilling the requirements for the degree of Doctor of Philosophy.

The results contained in this thesis have not been submitted in part or full to any other University or Institute for the award of any degree or diploma.

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ABSTRACT

The present research is an unique attempt to model a piezoelectric energy harvester by Variational Asymptotic Method (VAM). Vibration based energy harvesting has been an exciting topic of research over the last decade. The main motivation behind this research is low power electronics and also utilization of daily unused energy. The major challenge in research of modeling of piezoelectric transducers lies in capturing the coupling effects both electromechanical coupling as well as structural coupling. Full scale 3D FEM based analysis is capable of doing a coupled field analysis but they are computationally expensive on the contrary reduced models like the present one, makes a balance of computational efficiency, simplicity and 3D level accuracy. Piezoelectric materials are preferred over all other materials because of their ease of application and large power densities. The most typical type of piezoelectric energy harvesters are bimorph type cantilevers which generates voltage output for a base excitation. The present work initiates with developing a theory based on Variational Asymptotic Method (VAM) for modeling a piezoelectric sensor under static and dynamic load. The three dimensional electrical as well as mechanical field variables are recovered and validated with the 3D FEM based solution. The validation shows the accuracy and efficiency of VAM based dimensional reduction for a multiphysics analysis like a sensor model. The theory was further extended to incorporate an independent electrical variable in the cross sectional model itself. Improved constitutive model provided a strong platform for modeling and analysis of an energy harvester. A bimorph cantilever energy harvester model with a tip mass was studied. Apart from being able to capture the electromechanical coupling correctly structural coupling was also captured, which is overlooked in most of the literatures.

Closed form expressions were developed for voltage, tip velocity, power and current. The response obtained were validated with the experimental work from past literatures. The initial harvester model was studied on Euler-Bernoulli assumption, which is quite valid for slender harvester model. For comparatively thick beam harvester or to capture the shear mode we need to model the harvester based on Timoshenko beam theory. Shear mode harvester shows a shear-bending coupling at the cross sectional level. Both transverse and shear harvesters were modelled and a comparative study was performed based on the theory. Shear mode harvester shows a better power output as compared to transverse mode. Optimisation of load resistance and damping ratio are some of the key factors which influence the performance of both type of energy harvesters. Some of the parametric studies such as material parameters and geometric parameters were performed to study the influence of them on performance of energy harvester. This gives an overall complete study on piezoelectric based energy harvester modeling.

सार

वर्तमान अनुसंधान, चरणीय अशांमटटिक विधि (वीएएम) द्वारा एक पीजोइलेक्ट्रिक ऊर्जा हारवेस्टर मॉडल के लिए एक अनूठा प्रयास है। कंपन आधारित ऊर्जा संचयन पिछले दशक में अनुसंधान का एक रोमांचक विषय रहा है। इस शोध के पीछे मुख्य प्रेरणा कम बिजली इलेक्ट्रॉनिक्स और दैनिक अप्रयुक्त ऊर्जा का उपयोग भी है। पोज़ेइलेक्ट्रिक ट्रांसड्यूसर के मॉडलिंग के शोध में प्रमुख चुनौती युग्मन प्रभाव दोनों इलेक्ट्रोमैकेनिक युग्लेशन और साथ ही स्ट्रक्चरल युग्मन पर कब्जा करने में है। पूर्ण पैमाने पर 3 डी एफईएम आधारित विश्लेषण एक युग्मित क्षेत्र विश्लेषण करने में सक्षम है, लेकिन वर्तमान में इसकी तुलना में विपरीत मॉडल पर कम्प्यूटेशनल महंगा हैं, कम्प्यूटेशनल दक्षता, सादगी और 3 डी स्तर की सटीकता का संतुलन बनाते हैं। अन्य सभी सामग्रियों से पीएजएइलेक्ट्रिक सामग्री को पसंद किया जाता है क्योंकि उनके आवेदन में आसानी और बड़ी शक्ति घनत्व है। सबसे सामान्य प्रकार के पीजोइलेक्ट्रिक ऊर्जा हार्वेस्टर ब्योमोरफ़ प्रकार ब्रैकट है जो बेस उत्तेजना के लिए वोल्टेज आउटपुट उत्पन्न करता है। वर्तमान काम स्थिर और गतिशील लोड के तहत एक पीजोइलेक्ट्रिक सेंसर मॉडलिंग के लिए वैरिएशन असिम्पटोटिक विधि (वीएएम) के आधार पर एक सिद्धांत विकसित करने के साथ शुरू होता है। तीन आयामी विद्युत और साथ ही मैकेनिकल फ़ील्ड वैरिएबल 3 एफआईएम आधारित समाधान के साथ पुनर्प्राप्त और मान्य हैं। मान्यता सेंसर मॉडल की तरह मल्टीफेजरी विश्लेषण के लिए VAM आधारित आयामी कमी की सटीकता और दक्षता को दर्शाती है। इस सिद्धांत को आगे क्रॉस अनुभागीय मॉडल में स्वतंत्र विद्युत वैरिएबल को शामिल करने के लिए आगे बढ़ा दिया गया था। सुधारित गठित मॉडल ने एक ऊर्जा हार्वेस्टर के मॉडलिंग और विश्लेषण के लिए एक मजबूत मंच प्रदान किया। टिप द्रव्यमान के साथ एक बियरोफ ब्रैकट ऊर्जा हार्वेस्टर मॉडल का अध्ययन किया गया। इलेक्ट्रोमैकेनिक युग्लेशन को सही ढंग से संरचनात्मक युग्मन करने में सक्षम होने के अलावा भी कब्जा कर लिया गया था, जो कि अधिकांश साहित्य में अनदेखी की गई है.

बंद फार्म अभिव्यक्ति वोल्टेज, टिप वेग, शक्ति और वर्तमान के लिए विकसित किए गए थे। प्राप्त की गई प्रतिक्रिया पिछले लेखों से प्रयोगात्मक कार्य के साथ मान्य थी। प्रारंभिक हारवेस्टर मॉडल का अध्ययन यूलर-बर्नोली धारणा पर किया गया था, जो पतला हारवेस्टर मॉडल के लिए काफी मान्य है। अपेक्षाकृत मोटी बीम हार्वेस्टर के लिए या कतरनी मोड पर कब्जा करने के लिए हमें टिमोशेनको किरण सिद्धांत के आधार पर हारवेस्टर के मॉडल की आवश्यकता है। कतर मोड मोडवेस्टर क्रॉस अनुभागीय स्तर पर एक कतरनी-झुकने युग्लेशन दिखाता है। दोनों अनुप्रस्थ और कतरनी हार्वेस्टर मॉडल किए गए थे और एक तुलनात्मक अध्ययन सिद्धांत के आधार पर किया गया था। अनुप्रस्थ मोड के मुकाबले शियर मोड हारवेस्टर बेहतर विद्युत उत्पादन को दर्शाता है। भार प्रतिरोध और भिगोना अनुपात का अनुकूलन कुछ

प्रमुख कारक हैं जो दोनों प्रकार के ऊर्जा वाले कटाई के प्रदर्शन को प्रभावित करते हैं। कुछ पैरामीट्रिक अध्ययन जैसे भौतिक पैरामीटर और ज्यामितीय

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NOMENCLATURES

$\mathbf{\Gamma}$	Second order 3D strain tensor
ε	One dimensional strain
$\mathbf{E}$	Electric field
$\tilde{\phi}$	3D Electric potential
ϕ	1D electrical variable, function of x_1 only
r	Reference line
$\overline{S}$	Reference cross section
χ	Coordinate curves
$\mathbf{b}_i$	Orthonormal triad for undeformed cross section
$\mathbf{B}_i$	Orthonormal triad for deformed cross section
$\hat{\mathbf{r}}$	Position vector of a material point in undeformed cross section
$\mathbf{r}$	Locus of centroid of the cross section
$\mathbf{C}^{Bb}$	Rotation tensor
$\mathbf{R}$	Position vector of that material point whose 3D location is average of the locations of all material point constituting cross section
$\hat{\mathbf{R}}$	Position vector of a material point in deformed cross section
$\hat{w}_i$	Warping field

$\mathbf{g}_i$	Three dimensional undeformed covariant base vector
$\mathbf{G}_i$	Three dimensional deformed covariant base vector
$\mathbf{g}^i$	Three dimensional undeformed contravarient base vector
$\mathbf{G}_i$	Three dimensional deformed contravarient base vector
$\mathbf{F}_{ij}$	Mixed basis component of deformation gradient tensor
$\boldsymbol{\gamma}$	One dimensional axial strain
$\boldsymbol{\kappa}_i$	Moment strains
$\tilde{\mathbf{K}}$	Curvature vector of the deformed beam
$\tilde{\mathbf{k}}$	Curvature of the undeformed beam
k_1	Initial twist
H	Electromechanical enthalpy
$\mathbf{C}_{\mathrm{E}}$	Fourth order material tensor measured at constant electric field
$\tilde{\mathbf{e}}^{\mathrm{a}}$	Third order electro mechanical coupling tensor
χ^{s}	Second order permittivity tensor measured
K	Kinetic energy
$\tilde{\mathbf{u}}$	Absolute velocity of generic point
$\boldsymbol{\Omega}$	Inertial angular velocity
V	Discretised form of the warping field
η	Mass per unit length
$\eta\xi$	First distributed mass moment of inertia
Υ	Second distributed mass moment of inertia

$\boldsymbol{\sigma}$	Second order three dimensional stress tensor
$\bar{W}$	Virtual work
h	Characteristic size of the cross section
l	Characteristic wavelength of axial deformation
μ	Order of elastic constant
S^u	Element shape function
n	Number of nodes of an element
Ψ_1	Null space matrix
Λ	Lagrange multiplier
S	Stiffness matrix
f^a	Actuation force vector
B	Strain displacement matrix
c_p	Capacitance per unit length
q	Charge
M	Mass matrix
$\tilde{C}$	Damping matrix
ϑ	System coupling matrix
R	Load resistance